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$\frac{1}{N}$ expansion of the nonequilibrium infinite-U Anderson model

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Results are presented for the nonequilibrium infinite-U Anderson model using a large N approach, where N is the degeneracy of the impurity level, and where nonequilibrium is established by coupling the level to two leads at two different chemical potentials so that there is current flow. A slave-boson representation combined with Keldysh functional integral methods is employed. Expressions for the static spin susceptibility χ_S and the conductance G are presented to $\mathcal{O}(\frac{1}{N})$ and for an applied voltage difference V less than the Kondo temperature. The correlation function for the slave boson is found to be significantly modified from its equilibrium form in that it acquires a rapid decay in time with a rate that equals the current-induced decoherence rate. Physical observables are found to have a rather complex dependence on the coupling strength to the two leads which can lead to asymmetric behavior $\chi_S(V)\neq\chi_S(-V)$, $G(V)\neq G(-V)$ both in the mixed valence and in the Kondo regime.

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I. INTRODUCTION

The theoretical problem of strong correlations coupled with nonequilibrium has become an active area of research in recent years, in part due to the enormous success in realizing experimental systems which can be driven out of equilibrium in a controlled manner. Some examples of these are current-carrying quantum dots and single-molecule devices,[1] strongly driven ferromagnetic systems,[2] and cold atoms trapped in optical lattices with rapidly tunable parameters.[3] One of the theoretical challenges in the study of out-of-equilibrium strongly correlated systems is that, unlike systems in equilibrium which are characterized by some underlying principles such as the energy minimization principle, no basic underlying principles are known for out-of-equilibrium systems making it rather difficult to develop general theoretical techniques to study them.

Perhaps the most actively studied out-of-equilibrium systems are nonequilibrium quantum-impurity models which are systems characterized by a few local degrees of freedom coupled to one or more reservoirs (as in a quantum dot or a molecular conductor), and where nonequilibrium is achieved by maintaining the reservoirs at different chemical potentials and/or by subjecting the system to time-dependent fields. For strong local interactions the ground state of quantum-impurity models show many-body resonances such as the Kondo or polaronic resonance. The effect of current flow on these resonances has been studied using a variety of methods such as renormalized perturbation theory,[4] flow equation methods,[5] real-time renormalization group on the Keldysh contour,[6,7] and functional renormalization-group methods.[8] While these approaches are applicable when the external drive is large as compared to the Kondo temperature, in the opposite limit of drive small compared to the Kondo temperature, perturbative methods based on Fermi-liquid theory have been used.[9] There have also been efforts at developing exact solutions based on the construction of exact scattering states in the presence of current flow.[10,11] There are also several promising numerical methods that are being developed such as the real-time numerical renormalization-group method,[12] quantum Monte Carlo computation of real-time Keldysh diagrams,[13,14] iterative summation of real-time path integrals,[15] and the imaginary time formulation of real-time nonequilibrium problems.[16]

In this paper we will use large-N methods[17] to study a nonequilibrium quantum-impurity model. In particular, we will study the Anderson model when the on-site Coulomb interaction $U=\infty$, and in addition the system has been driven out of equilibrium due to current flow. N here will represent the degeneracy of the impurity level. The physical systems this corresponds to are quantum dots or molecular devices where the level active in transport is characterized by a total angular momentum $J=L+S$ which is large, and hence has a large degeneracy $N=2J+1$. This could arise due to the particular form of the confining potential in the quantum dot, or by the use of a molecule where conduction occurs via a metal ion with a partially empty outermost d or f orbital. Note that the infinite-U Anderson model under out-of-equilibrium conditions has so far been studied using the noncrossing approximation (NCA)[18] and slave-boson mean-field methods.[19] In this paper we will also employ the slave-boson representation which is a convenient way to project out all states except the empty and singly occupied state of the dot.[20] However, we will go beyond mean field by including the effect of fluctuations to $\mathcal{O}(\frac{1}{N})$. Our theoretical approach is closest to that of Read *et al.*,[21,22] but carried out for a nonequilibrium system using Keldysh functional integral methods.

A few words on the regime of validity of the results presented in this paper. The $U=\infty$ limit of the Anderson model is the so-called mixed-valence regime where the system is characterized by both local charge as well as spin fluctuations. The Kondo regime may be accessed by making the bare level energy large and negative in which case the charge fluctuations are frozen out and only the spin fluctuations exist. This limit can be taken in a straightforward way in all physical observables. Thus we will present results for the nonequilibrium static spin susceptibility and the conductance in both the mixed valence as well as in the Kondo regime.

This paper is organized as follows. In Sec. II we present the model, and write it as a Keldysh path integral suitable for studying nonequilibrium systems. In Sec. III we briefly present the main results of the paper before turning to the full calculation. In Sec. IV we study the Keldysh path integral in the limit of $N\to\infty$ when a mean-field or saddle-point approximation becomes exact. In this limit the voltage dependence of the local charge density, static susceptibility, and the conductance are derived. Following this, the rest of the paper is devoted to the study of the effect of fluctuations to $\mathcal{O}(1/N)$. As found by Read *et al.*,[21] the $1/N$ corrections are in general associated with infrared divergences whose origin is the zero mode of the slave-boson representation. While the infrared divergences are logarithmic in equilibrium, we find that out of equilibrium the divergences become more severe with a pole structure. However, just as in equilibrium, in the computation of all physical observables these infrared divergences are found to cancel so that the final expressions are well defined.

The $\mathcal{O}(1/N)$ computation is organized as follows. In Sec. V the mean-field saddle-point expressions for the level position and the level broadening are corrected to $\mathcal{O}(1/N)$. In Sec. VI the local impurity charge density is computed. In Sec. VII the bosonic correlation function is evaluated. While in equilibrium the bosonic correlation function has a power-law decay in time with an exponent consistent with x-ray edge physics,[21] for the current carrying case we find that the long-time behavior has both a power law as well as a rapid exponential decay in time, with the latter arising due to current-induced decoherence. The bosonic correlation function appears in the computation of various physical observables. We present results for the static susceptibility in Sec. VIII, while expressions for the impurity spectral density and conductance are presented in Sec. IX. Many of the details of the computation are relegated to the appendices. Finally we conclude in Sec. X.

II. MODEL

We use the slave-boson representation[17] of the infinite-U Anderson model which is a convenient way to project out all except the empty and singly-occupied states of the impurity level. The Hamiltonian in this representation is

$$H = \sum_m E_0 f_m^\dagger f_m + \sum_{k,m,\alpha} \epsilon_k c_{km\alpha}^\dagger c_{km\alpha} + \frac{1}{\sqrt{N}} \sum_{k,m,\alpha=L,R} V_\alpha (c_{km\alpha}^\dagger f_m b^\dagger + f_m^\dagger c_{km\alpha} b), \tag{1}$$

where $m=-J,\ldots,J$ represents the spin projection of the local level, $N=2J+1$ is the degeneracy of the level, $c_{km\alpha}$ represent the lead electrons, and we have generalized the case where there are two leads (labeled by $\alpha=L,R$) which will be maintained at two different chemical potentials $\mu_{L,R}$ to capture the nonequilibrium current-carrying case. $\frac{V_{\alpha=L,R}}{\sqrt{N}}$ is the hybridization to the two leads. The above Hamiltonian is accompanied by the constraint

$$1 = \sum_m f_m^\dagger f_m + b^\dagger b \tag{2}$$

to ensure that the system remains within the restricted Hilbert space of an empty or singly occupied local level.

We write the Keldysh path integral[23] for Eq. (1) and impose the constraint in Eq. (2) by introducing two Lagrange multipliers $\lambda_\pm$

$$Z_K = \int \mathcal{D}[f_{m\pm},\bar{f}_{m\pm},\lambda_\pm,b_\pm,b_\pm^*,c_m,\bar{c}_m]\exp(i\mathrm{Tr}[S_K]), \tag{3}$$

where the Tr symbol in Eq. (3) represents a trace over time indices, and

$$\begin{aligned} S_K = {} & \sum_m (\bar{f}_{m-} \;\; \bar{f}_{m+}) \begin{pmatrix} i\partial_t - E_0 & 0 \\ 0 & i\partial_t - E_0 \end{pmatrix} \begin{pmatrix} f_{m-} \\ f_{m+} \end{pmatrix} \\ & + \sum_{km\alpha} (\bar{c}_{km\alpha-} \;\; \bar{c}_{km\alpha+}) g_{c\alpha}^{-1} \begin{pmatrix} c_{km\alpha-} \\ c_{km\alpha+} \end{pmatrix} \\ & + \sum_{km\alpha} (\bar{f}_{m-} \;\; \bar{f}_{m+}) \begin{pmatrix} -\frac{V_\alpha}{\sqrt{N}} b_- & 0 \\ 0 & \frac{V_\alpha}{\sqrt{N}} b_+ \end{pmatrix} \begin{pmatrix} c_{km-} \\ c_{km+} \end{pmatrix} \\ & + \sum_{km\alpha} (\bar{c}_{km-} \;\; \bar{c}_{km+}) \begin{pmatrix} -\frac{V_\alpha}{\sqrt{N}} b_-^* & 0 \\ 0 & \frac{V_\alpha}{\sqrt{N}} b_+^* \end{pmatrix} \begin{pmatrix} f_{m-} \\ f_{m+} \end{pmatrix} \\ & + (b_-^* \;\; b_+^*) \begin{pmatrix} i\partial_t & 0 \\ 0 & i\partial_t \end{pmatrix} \begin{pmatrix} b_- \\ b_+ \end{pmatrix} - \lambda_- \left[\sum_m \left(\bar{f}_{m-} f_{m-} + \frac{1}{2} \right) \right. \\ & \left. + b_-^* b_- - 1 \right] + \lambda_+ \left[\sum_m \left(\bar{f}_{m+} f_{m+} + \frac{1}{2} \right) + b_+^* b_+ - 1 \right]. \end{aligned} \tag{4}$$

In the above $g_{c\alpha}^{-1}$ is the inverse Green's function for the leads and is a 2×2 matrix in Keldysh space. It is convenient to integrate out the lead electrons to obtain

$$\begin{aligned} S_K = {} & \sum_m (\bar{f}_{m-} \;\; \bar{f}_{m+}) \begin{pmatrix} i\partial_t - E_0 - \lambda_- & 0 \\ 0 & i\partial_t - E_0 + \lambda_+ \end{pmatrix} \begin{pmatrix} f_{m-} \\ f_{m+} \end{pmatrix} \\ & + (b_-^* \;\; b_+^*) \begin{pmatrix} i\partial_t - \lambda_- & 0 \\ 0 & i\partial_t + \lambda_+ \end{pmatrix} \begin{pmatrix} b_- \\ b_+ \end{pmatrix} \\ & - \frac{1}{N} \sum_{m\alpha} V_\alpha^2 (\bar{f}_{m-} \;\; \bar{f}_{m+}) \begin{pmatrix} b_- & 0 \\ 0 & -b_+ \end{pmatrix} \begin{pmatrix} g_{--}^{c,\alpha} & g_{-+}^{c,\alpha} \\ g_{+-}^{c,\alpha} & g_{++}^{c,\alpha} \end{pmatrix} \\ & \times \begin{pmatrix} b_-^* & 0 \\ 0 & -b_+^* \end{pmatrix} \begin{pmatrix} f_{m-} \\ f_{m+} \end{pmatrix} + (\lambda_- - \lambda_+) \left(1 - \frac{N}{2} \right). \end{aligned} \tag{5}$$

Performing a rotation to retarded (R), advanced (A), Keldysh (K) space,[23] and defining the quantum fields as $O_q=(O_- - O_+)/2$ and the classical field as $O_{\mathrm{cl}}=(O_- + O_+)/2$, we get

$$S_K = \sum_m (\bar{f}_{m,q} \quad \bar{f}_{m,\mathrm{cl}})[g_{0f}^{-1} - \lambda_{\mathrm{cl}}\tau_0 - \lambda_q\tau_x - (b_{\mathrm{cl}}\tau_0 + b_q\tau_x)\Sigma_c(b^*_{\mathrm{cl}}\tau_0 + b^*_q\tau_x)]\begin{pmatrix} f_{m,\mathrm{cl}} \\ f_{m,q}\end{pmatrix} + 2(b^*_{\mathrm{cl}} \quad b^*_q) \times \begin{pmatrix} -\lambda_q & i\partial_t - \lambda_{\mathrm{cl}} \\ i\partial_t - \lambda_{\mathrm{cl}} & -\lambda_q \end{pmatrix}\begin{pmatrix} b_{\mathrm{cl}} \\ b_q \end{pmatrix} + 2\lambda_q\left(1 - \frac{N}{2}\right), \quad (6)$$

where the Σ_c are the self-energies due to coupling to leads,

$$\Sigma_c = \begin{pmatrix} \Sigma_c^R & \Sigma_c^K \\ 0 & \Sigma_c^A \end{pmatrix} \quad (7)$$

with $\Sigma_c^{i=R,A,K}(t,t') = \frac{1}{N}\Sigma_{k,\alpha=L,R} V_\alpha^2 g_c^{i=R,A,K}(k;t,t')$. Thus the self-energies due to coupling to leads is $\mathcal{O}(\frac{1}{N})$. We will make the assumption of constant density of states in the leads which gives

$$\Sigma_c^R = -\frac{i}{N}\pi\rho\sum_{\alpha=L,R} V_\alpha^2 = -i(\Gamma_L + \Gamma_R) = -i\Gamma, \quad (8)$$

$$\Sigma_c^A = i\Gamma, \quad (9)$$

$$\Sigma_c^K = -2i\Gamma\sum_{\alpha=L,R}\frac{\Gamma_\alpha}{\Gamma}[1 - 2f(\omega - \mu_\alpha)]. \quad (10)$$

The aim will be to use the action in Eq. (5) to evaluate physical observables perturbatively in $1/N$. Before turning to the full computation, we present the main results in the next section.

III. BRIEF DISCUSSION OF RESULTS

Let us suppose that the chemical potential of the left lead is $\mu_L = V/2$ while that of the right lead is $\mu_R = -V/2$. As specified in Eq. (8), let $\Gamma_L(\Gamma_R)$ be the self-energy due to coupling to the left (right) lead, while $\Gamma = \Gamma_L + \Gamma_R$ is the total self-energy. In terms of the above parameters, the static susceptibility in the Kondo regime (denoted by the superscript $n_F = 1$ to indicate the value of the charge on the impurity level) is found to have the following universal form:

$$\chi_S^{n_F=1} = \frac{g^2\mu_B^2 J(J+1)}{3T_K}\left[1 + 1.5\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_K}\right)^2 + \frac{1}{N}\left(\frac{\Gamma_L - \Gamma_R}{\Gamma}\right)\left(\frac{V}{2T_K}\right)C_{S1} + \frac{1}{N}\left(\frac{V}{2T_K}\right)^2(C_{S2} + C_{S3} - C_{S1}) - \frac{1}{N}(4.5 + 3C_{S0} + C_{S3} - C_{S1})\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_K}\right)^2\right], \quad (11)$$

where $T_K = T_A^0(1 - \frac{C_{S0}}{N})$ is the Kondo temperature correct to $\mathcal{O}(1/N)$, with $T_A^0 = De^{\pi E_0/N\Gamma}$ being the mean-field Kondo temperature,[22] and the C_{Si} are numbers specified in the text [after Eq. (146)]. Thus one finds that for an asymmetric coupling to leads $(\Gamma_L \neq \Gamma_R)$, $\chi_S(V) \neq \chi_S(-V)$. This lack of symmetry when $V \leftrightarrow -V$ arises due to the fermi-level dependence of the Kondo temperature. To see this we set the coupling to one of the leads (say Γ_R) to zero. This corresponds to an equilibrium configuration where there is no current flow. For this case Eq. (11) reduces to

$$\chi_S^{n_F=1}(\mu_L = V/2, \Gamma_R = 0) = \frac{g^2\mu_B^2 J(J+1)}{3T_K}\left[1 + \frac{1}{N}\left(\frac{V}{2T_K}\right)C_{S1} + \frac{1}{N}\left(\frac{V}{2T_K}\right)^2(C_{S2} + C_{S3} - C_{S1})\right]. \quad (12)$$

Thus the terms in Eq. (12) can be interpreted as a change in the Kondo temperature arising from a change in the chemical potential of the left lead by $\delta\mu_L = V/2$. The asymmetry $\chi_S(V) \neq \chi_S(-V)$ in the Kondo regime therefore arises when the level is unequally coupled to two leads, each associated with a different equilibrium Kondo temperature. In contrast, the terms of the type $\frac{\Gamma_L\Gamma_R}{\Gamma^2}(\frac{V}{T_K})^2$ in Eq. (11) are purely nonequilibrium terms that arise due to inelastic-scattering processes in the energy window V when there is current flow, and are thus associated with current-induced decoherence. The identification of these terms with decoherence becomes clearer below when we discuss the slave-boson correlation function.

We now turn to the discussion of the conductance. Here too one finds that the fermi-level dependence of the spectral density can give rise to a conductance that is asymmetric under $V \rightarrow -V$.[24] In particular the mean-field saddle-point expression for the conductance in the mixed-valence regime is found to be

$$G_{\mathrm{sp}}(V) = G_{\mathrm{sp}}(V=0)\left[1 - \left(\frac{\Gamma_L - \Gamma_R}{\Gamma}\right)\left(\frac{2V}{T_A^0}\right) - \frac{12\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2 + 3\left(\frac{V}{T_A^0}\right)^2 - 3n_F\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2\right], \quad (13)$$

where n_F is the charge density on the level when $\mu_L = \mu_R = 0$, and $G_{\mathrm{sp}}(V=0) = \frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}(\frac{\pi n_F}{N})^2$. The conductance in the Kondo regime can be accessed by taking the limit $n_F \rightarrow 1$ in Eq. (13). Thus for a symmetric coupling to the two leads, the mean-field conductance in the Kondo regime becomes

$$G_{\mathrm{sp}}^{n_F=1}(V;\Gamma_L = \Gamma_R) = G_{\mathrm{sp}}^{n_F=1}(V=0)\left[1 - \frac{3}{4}\left(\frac{V}{T_A^0}\right)^2\right]. \quad (14)$$

The $1/N$ correction to the conductance for the case of symmetric couplings to leads is given in Eq. (162) for the mixed-valence regime and in Eq. (166) in the Kondo regime.

We now turn to the discussion of the bosonic correlation function $\bar{D}_K(t,t') = -i\langle\{b(t), b^\dagger(t')\}\rangle$ which is used to obtain the physical observables discussed above. At the mean-field level, $b(t) \rightarrow \langle b\rangle$ in the Hamiltonian [Eq. (1)], so that the $U(1)$ symmetry of the Hamiltonian is broken. In equilibrium, including fluctuations to $\mathcal{O}(1/N)$, the correlation function becomes[21,22]

$$\bar{D}^K_{\rm eq}(t) = -2i(1-n_F)\left(1-\frac{n_F^2}{N}\ln(tT_A^0)\right). \tag{15}$$

It was argued that[21,22] since the model in Eq. (1) cannot have any broken-symmetry state, including terms to higher orders in $1/N$ should lead to a power-law decay in the bosonic correlation function so that the symmetry of the Hamiltonian is restored. Thus,

$$\bar{D}^K_{\rm eq}(t) \sim \frac{1}{(tT_A^0)^{\alpha}}, \tag{16}$$

where $\alpha = N\frac{\delta^2}{\pi^2} = \frac{n_F^2}{N}$ and equals the Nozieres-de Dominicis infrared exponent for the response of an electron gas subjected to a sudden change in potential.[25]

We find that the result for the bosonic correlation function for the current carrying case, and for long times ($Vt \gg 1$) is

$$\bar{D}^K_{\rm neq}(t) = -2i(1-n_F)\left(1-\frac{c_L+c_R}{N}\ln(tT_A^0)-\frac{c_{\rm dec}}{N}Vt\right) \tag{17}$$

where the coefficients $c_{L,R,\rm dec}$ are specified in Eqs. (125) and (126). The c_i are weakly voltage dependent and neglecting terms of $\mathcal{O}(V/T_A^0)$ are

$$c_{L,R} = n_F^2\frac{\Gamma_{L,R}^2}{\Gamma^2} + \mathcal{O}\left(\frac{V}{T_A^0}\right), \tag{18}$$

$$c_{\rm dec} = n_F^2\frac{2\Gamma_L\Gamma_R}{\Gamma^2} + \mathcal{O}\left(\frac{V}{T_A^0}\right). \tag{19}$$

For evaluating quantities to $\mathcal{O}(1/N)$, Eq. (17) is sufficient. However it is interesting to consider how $\bar{D}^K$ would change when higher order in $1/N$ terms are included. Following Eq. (16), we expect that the bosonic correlation function will have the form

$$\bar{D}^K_{\rm neq}(t) \sim \frac{1}{(tT_A^0)^{\alpha_{\rm neq}}}\exp\left(-\frac{n_F^2}{N}\frac{2\Gamma_L\Gamma_R}{\Gamma^2}Vt\right), \tag{20}$$

where $\alpha_{\rm neq} = (c_L+c_R)/N$. Thus, to all orders in $1/N$ the bosonic correlation function will be characterized with a long-time power-law decay along with rapid exponential decay in time, the latter arising due to current-induced decoherence. The rate of decoherence is $\frac{\Gamma_L\Gamma_R}{\Gamma^2}V$, and is an energy scale that appears repeatedly in all physical observables. Note that Eq. (20) is also consistent with nonequilibrium x-ray edge physics, i.e., the response of an out-of-equilibrium electron gas to a sudden change in potential studied recently in various contexts.[26]

We now turn to the derivation of the above results.

IV. MEAN-FIELD SADDLE-POINT TREATMENT

In the mean-field saddle-point treatment, one assumes the fields $b_{{\rm cl},q}, \lambda_{{\rm cl},q}$ in Eq. (6) to be constants in time. The action S_K is then minimized both with respect to the classical fields $b_{\rm cl}, \lambda_{\rm cl}$ and the quantum fields b_q, λ_q. The classical saddle points $\frac{\delta S_K}{\delta\lambda_{\rm cl}}=0, \frac{\delta S_K}{\delta b_{\rm cl}}=0$ are automatically satisfied for $b_q = \lambda_q = 0$. Thus, in order to satisfy the saddle-point equations with respect to the quantum fields $\frac{\delta S_K}{\delta\lambda_q}=0, \frac{\delta S_K}{\delta b_q}=0$ it is sufficient to expand S_K to linear order in the quantum field. To carry these steps out, we integrate out the fermionic fields in Eq. (6) to obtain

$$S_K = -iN{\rm Tr}\ \ln[G_{\rm mf}^{-1} - \lambda_q\tau_x - b_{\rm cl}\Sigma_c b_q\tau_x - b_q\tau_x\Sigma_c b_{\rm cl} + O(b_q^2)] + 2\lambda_q\left(1-\frac{N}{2}\right) - 2\lambda_q(b_q^2+b_{\rm cl}^2) - 4\lambda_{\rm cl}b_{\rm cl}b_q, \tag{21}$$

where the mean-field fermionic Green's function is

$$G_{\rm mf}^{-1} = g_{0f}^{-1} - \lambda_{\rm cl} - b_{\rm cl}^2\Sigma_c \tag{22}$$

Defining

$$\tilde{\Gamma} = b_{\rm cl}^2\Gamma, \tag{23}$$

where $\tilde{\Gamma}$ plays the role of the level broadening,

$$G^R_{\rm mf}(\omega) = \frac{1}{\omega - E_0 - \lambda_{\rm cl} + i\tilde{\Gamma}}, \tag{24}$$

$$G^K_{\rm mf}(\omega) = \left(\frac{\tilde{\Gamma}}{\Gamma}\right)G^R_{\rm mf}(\omega)\Sigma^K_c(\omega)G^A_{\rm mf}(\omega). \tag{25}$$

From Eq. (21), the saddle-point equation for λ_q, $\frac{\delta S_K}{\delta\lambda_q}=0$ gives

$$2\left(1-\frac{N}{2}\right) - 2b_{\rm cl}^2 - iN\left[\frac{\partial}{\partial\lambda_q}{\rm Tr}\ \ln(G_{\rm mf}^{-1}-\lambda_q\tau_x)\right]_{\lambda_q=0} = 0. \tag{26}$$

This leads to

$$1-\frac{N}{2} = b_{\rm cl}^2 - \frac{iN}{2}{\rm Tr}[G^K_{\rm mf}]. \tag{27}$$

Using Eq. (25) the above becomes,

$$1 = b_{\rm cl}^2 + N\int\frac{d\omega}{2\pi}\frac{2\Gamma b_{\rm cl}^2}{(\omega-E_0-\lambda_{\rm cl})^2+\Gamma b_{\rm cl}^2}\left(\frac{\Gamma_L}{\Gamma}f(\omega-\mu_L) + \frac{\Gamma_R}{\Gamma}f(\omega-\mu_R)\right). \tag{28}$$

After performing the frequency integrations, we obtain

$$1 = \frac{\tilde{\Gamma}}{\Gamma} + \frac{N}{\pi}\left[\frac{\Gamma_L}{\Gamma}\arctan\frac{\tilde{\Gamma}}{E_0+\lambda_{\rm cl}-\mu_L} + \frac{\Gamma_R}{\Gamma}\arctan\frac{\tilde{\Gamma}}{E_0+\lambda_{\rm cl}-\mu_R}\right]. \tag{29}$$

Similarly, minimizing Eq. (21) with respect to $b_{q,\rm cl}$ leads to

$$-4\lambda_{\rm cl}b_{\rm cl} - iN\left[\frac{\partial}{\partial b_q}{\rm Tr}\ \ln(G_{\rm mf}^{-1} - b_{\rm cl}\Sigma_c b_q\tau_x - b_q\tau_x\Sigma_c b_{\rm cl})\right]_{b_q=0} = 0. \tag{30}$$

Using expressions for Σ_c, the above leads to

$$\frac{\lambda_{cl}}{\Gamma} + N\int \frac{d\omega}{2\pi}[G^R_{mf}(\omega) + G^A_{mf}(\omega)]\left[\frac{\Gamma_L}{\Gamma}f(\omega-\mu_L) + \frac{\Gamma_R}{\Gamma}f(\omega-\mu_R)\right] = 0, \tag{31}$$

which after performing the frequency integrations gives

$$\frac{\lambda_{cl}}{\Gamma} + \frac{N}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\frac{\sqrt{(\mu_L - E_0 - \lambda_{cl})^2 + \tilde{\Gamma}^2}}{D} + \frac{\Gamma_R}{\Gamma}\ln\frac{\sqrt{(\mu_R - E_0 - \lambda_{cl})^2 + \tilde{\Gamma}^2}}{D}\right] = 0. \tag{32}$$

We will now proceed to solve the two saddle-point equations (29) and (32), and use the solution to evaluate various observables. The results obtained will be exact in the limit $N \to \infty$.

A. Solution of the saddle-point equations

Let us define

$$\lambda_{cl} + E_0 = \epsilon_F, \tag{33}$$

where ϵ_F is the effective position of the impurity level. When $N\to\infty$, $\Gamma, \tilde{\Gamma}\to 0$, and $N\Gamma$=const. Using this, Eq. (29) may be simplified to

$$1 = \frac{\tilde{\Gamma}}{\Gamma} + \frac{N\tilde{\Gamma}}{\pi}\left[\frac{\Gamma_L/\Gamma}{\epsilon_F - \mu_L} + \frac{\Gamma_R/\Gamma}{\epsilon_F - \mu_R}\right], \tag{34}$$

while Eq. (32) becomes (defining $\epsilon_F = T_A$ as the position of the level in the limit $N\to\infty$)

$$T_A = E_0 - \frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\frac{|T_A-\mu_L|}{D} + \frac{\Gamma_R}{\Gamma}\ln\frac{|T_A-\mu_R|}{D}\right]. \tag{35}$$

Let us define

$$m = \frac{N\Gamma}{\pi T_A}, \tag{36}$$

$$m_V = m\left[\frac{\Gamma_L/\Gamma}{1-\mu_L/T_A} + \frac{\Gamma_R/\Gamma}{1-\mu_R/T_A}\right]. \tag{37}$$

m should not to be confused with the label for the spin projection. Note that in equilibrium, $\mu_L = \mu_R = 0$ and $m_V = m$. In terms of these variables, Eq. (34) implies the following for the saddle-point solution for b_{cl}:

$$b_{cl}^2 = b_{sp}^2 = \frac{\tilde{\Gamma}}{\Gamma} = \frac{1}{1+m_V}, \tag{38}$$

whereas the impurity charge density is

$$n_F = -i\frac{N}{2}\mathrm{Tr}[G^K_{mf}] + \frac{N}{2} = 1 - \frac{\tilde{\Gamma}}{\Gamma} = \frac{m_V}{1+m_V}. \tag{39}$$

Note that in the Kondo limit, $m_V \gg 1$ so that $n_F \to 1$.

B. Solution for T_A

We solve Eq. (35) when $-E_0 \gg T_A$. Writing

$$T_A = T_A^0 + \delta T_A, \tag{40}$$

where

$$T_A^0 = De^{-\pi|E_0|/N\Gamma} \tag{41}$$

is the equilibrium solution for the impurity level, Eq. (35) becomes

$$|T_A^0 + \delta T_A - \mu_L|^{\Gamma_L/\Gamma}|T_A^0 + \delta T_A - \mu_R|^{\Gamma_R/\Gamma} = T_A^0. \tag{42}$$

For small voltages, $|\delta T_A|, |\mu_{L,R}| \ll T_A^0$, a Taylor expansion leads to the following expression for the change in T_A due to bias:

$$\delta T_A = \left(\frac{\Gamma_L\mu_L}{\Gamma} + \frac{\Gamma_R\mu_R}{\Gamma}\right) + \frac{1}{2T_A^0}\frac{\Gamma_L\Gamma_R}{\Gamma^2}(\mu_L-\mu_R)^2. \tag{43}$$

C. Mean-field impurity susceptibility

We now turn to the evaluation of the voltage dependence of the impurity susceptibility. The spin-response function at the mean-field level is given by

$$\chi^R_{mf}(\Omega) = \frac{i}{2}\sum_m (g\mu_B m)^2\int\frac{d\omega}{2\pi}[G^R_{mf}(\omega+\Omega)G^K_{mf}(\omega) + G^K_{mf}(\omega+\Omega)G^A_{mf}(\omega)]. \tag{44}$$

Using the identity $\sum_{m=-J\ldots J} m^2 = \frac{2J+1}{3}J(J+1) = \frac{N}{3}J(J+1)$, the spin susceptibility which is the zero-frequency spin-response function becomes

$$\chi_{sp} = \chi^R_{mf}(\Omega=0) = \frac{g^2\mu_B^2}{3}J(J+1)\left(\frac{N\tilde{\Gamma}}{\pi}\right)\left[\frac{\Gamma_L/\Gamma}{(T_A-\mu_L)^2+\tilde{\Gamma}^2} + \frac{\Gamma_R/\Gamma}{(T_A-\mu_R)^2+\tilde{\Gamma}^2}\right]. \tag{45}$$

For $N\to\infty$ we may drop terms of $\mathcal{O}(\tilde{\Gamma}^2)$,

$$\chi_{sp} = \frac{g^2\mu_B^2}{3}J(J+1)\frac{m}{(1+m_V)T_A}\sum_{i=L,R}\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^2}. \tag{46}$$

Taylor expanding Eq. (46) in powers of $\frac{\mu_{L,R}}{T_A^0}$ and defining

$$m_0 = \frac{N\Gamma}{\pi T_A^0}, \tag{47}$$

we find the following voltage dependence of the susceptibility at saddle point:

$$\chi_{sp} = \frac{g^2\mu_B^2}{3}J(J+1)\frac{m_0}{T_A^0(1+m_0)}\left[1 + \left(\frac{4+3m_0}{1+m_0}\right)\times\frac{\Gamma_L\Gamma_R}{2\Gamma^2}\left(\frac{\mu_L-\mu_R}{T_A^0}\right)^2 + \cdots\right]. \tag{48}$$

In the Kondo limit, $m_0 \gg 1$, or the equilibrium charge on the level $n_F=\frac{m_0}{1+m_0}\rightarrow 1$. In this case the static susceptibility becomes

$$\chi_{\rm sp}^{n_F=1} \rightarrow \frac{g^2\mu_B^2}{3T_A^0}J(J+1)\left[1+1.5\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\mu_L-\mu_R}{T_A^0}\right)^2+\cdots\right]. \tag{49}$$

D. Mean-field conductance

The current is given by[27]

$$I=\frac{ie}{\hbar}\frac{2\Gamma_L\Gamma_R}{\Gamma}\sum_m\int\frac{d\omega}{2\pi}[f(\omega-\mu_L)-f(\omega-\mu_R)] \times[G^R_{f_m,b}-G^A_{f_m,b}](\omega), \tag{50}$$

where

$$G^R_{f_m,b}(t,t')=-iT\langle b_-^\dagger(t)f_{m-}(t)f_{m-}^\dagger(t')b_-(t')\rangle - i\langle f^\dagger_{m+}(t')b_+(t')b_-^\dagger(t)f_{m-}(t)\rangle. \tag{51}$$

Within mean field, $b_\pm$ are constants in time and equal to the saddle-point value given in Eq. (38). Thus at zero temperature Eq. (50) becomes

$$I_{\rm mf}=\frac{Ne}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma}\left(\frac{\tilde\Gamma}{\Gamma}\right)\left[\tan^{-1}\left(\frac{\mu_L-T_A}{\tilde\Gamma}\right)-\tan^{-1}\left(\frac{\mu_R-T_A}{\tilde\Gamma}\right)\right]. \tag{52}$$

Let us set $\mu_L=eV/2,\mu_R=-eV/2$. The zero-bias conductance depends only on the equilibrium properties of the spectral density and is given by

$$G_{\rm sp}(V=0)=\left.\frac{\partial I_{\rm mf}}{\partial V}\right|_{V=0}=\frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}\frac{\tilde\Gamma^2}{T_A^2} =\frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\pi n_F(V=0)}{N}\right)^2. \tag{53}$$

The nonlinearity in the conductance arises due to the frequency and voltage dependence of the spectral density (namely, the voltage dependence of its position T_A and its width $\tilde\Gamma$). We find the following expression for the nonlinear conductance:

$$G_{\rm sp}(V)=\frac{\partial I_{\rm mf}}{\partial V}=G_{\rm sp}(V=0)\left[1-\left(\frac{\Gamma_L-\Gamma_R}{\Gamma}\right)\left(\frac{2V}{T_A^0}\right) -\frac{12\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2+3\left(\frac{V}{T_A^0}\right)^2-\frac{3m_0}{1+m_0}\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2\right]. \tag{54}$$

The above implies that for asymmetric coupling to the leads ($\Gamma_L\neq\Gamma_R$), the conductance shows a rectification-type behavior, i.e., $G_{\rm sp}(V)\neq G_{\rm sp}(-V)$. Whereas for symmetric couplings to the leads, the conductance reduces to

$$G_{\rm sp}(V;\Gamma_L=\Gamma_R)=G_{\rm sp}(V=0)\left[1-\frac{3}{4}\frac{m_0}{1+m_0}\left(\frac{V}{T_A^0}\right)^2\right]. \tag{55}$$

The conductance in the Kondo limit can be obtained by setting $m_0\gg 1$ in Eqs. (54) and (55). Thus, for symmetric couplings, we get

$$G_{\rm sp}^{n_F=1}(V;\Gamma_L=\Gamma_R)=G_{\rm sp}(V=0)\left[1-\frac{3}{4}\left(\frac{V}{T_A^0}\right)^2\right]. \tag{56}$$

The main results of this section are the expressions for the static susceptibility [Eqs. (48) and (49)], and the conductance [Eqs. (54)–(56)]. In the rest of the paper we will study how these results are modified when fluctuations to $\mathcal{O}(\frac{1}{N})$ are taken into account.

V. FLUCTUATIONS ABOUT MEAN FIELD

We now turn to the computation of how the saddle-point in Eqs. (29) and (32) gets modified when fluctuations are included. Formally the steps involved are to write $b_{\rm cl}\rightarrow b_{\rm sp}+b_{\rm cl}$, $b^*_{\rm cl}\rightarrow b_{\rm sp}+b^*_{\rm cl}$, $b_q\rightarrow\bar b_q+b_q$, and $b^*_q\rightarrow\bar b_q+b^*_q$. Then we integrate out all the fermionic and bosonic fields $b_{\rm cl},b^*_{\rm cl},b_q,b^*_q$, obtaining a resulting action that depends only on $S_K=S_K(b_{\rm sp},\bar b_q,\lambda_{\rm cl},\lambda_q)$. Each of the variables $x=b_{\rm sp},\bar b_q,\lambda_{\rm cl},\lambda_q$ are then determined by requiring that $\frac{\delta S_K(x,y\ldots)}{\delta x}=0$. Of course, the bosonic and fermionic fields cannot be integrated out exactly. This is therefore done perturbatively in $\frac{1}{N}$. Moreover, as discussed in Sec. IV, the saddle-point equations with respect to the classical fields $\frac{\delta S_K}{\delta b_{\rm sp}}=0,\frac{\delta S_K}{\delta\lambda_{\rm cl}}=0$ are always satisfied if all the quantum fields $\lambda_q=\bar b_q=0$. Thus to obtain the quantum saddle points, it suffices to expand S_K to only the leading power in the quantum fields $\lambda_q,\bar b_q$. To make the computation simple, we will carry this out separately for the saddle-point equation for λ and $b_{\rm sp}$.

A. Saddle-point equation for λ

In order to compute $1/N$ corrections to the saddle-point equation for λ [Eq. (29)], we write $b_{\rm cl}\rightarrow b_{\rm sp}+b_{\rm cl}$ and expand the action in Eq. (6) in powers of $b_{\rm cl},b_q$ and λ_q. To achieve this we first integrate out the fermion fields in Eq. (6) to obtain

$$\begin{aligned}S_K=&-iN\mathrm{Tr}\,\ln[G^{-1}_{\rm mf}-\lambda_q\tau_x-b_{\rm sp}\Sigma_c(b^*_{\rm cl}\tau_0+b^*_q\tau_x)-(b_{\rm cl}\tau_0\\&+b_q\tau_x)\Sigma_c b_{\rm sp}-(b_{\rm cl}\tau_0+b_q\tau_x)\Sigma_c(b^*_{\rm cl}\tau_0+b^*_q\tau_x)]\\&+2(b^*_{\rm cl}\ \ b^*_q)\begin{pmatrix}-\lambda_q & i\partial_t-\lambda_{\rm cl}\\ i\partial_t-\lambda_{\rm cl} & -\lambda_q\end{pmatrix}\begin{pmatrix}b_{\rm cl}\\ b_q\end{pmatrix}+2\lambda_q\left(1-\frac{N}{2}\right)\\&-2\lambda_q b^2_{\rm sp}-2\lambda_q b_{\rm sp}(b_{\rm cl}+b^*_{\rm cl})-2\lambda_{\rm cl}b_{\rm sp}(b_q+b^*_q).\end{aligned} \tag{57}$$

Let us define,

$$\tilde G^{-1}_{\rm mf}(\lambda_q)=G^{-1}_{\rm mf}-\lambda_q\tau_x. \tag{58}$$

The solution to the above equation to leading order in λ_q is

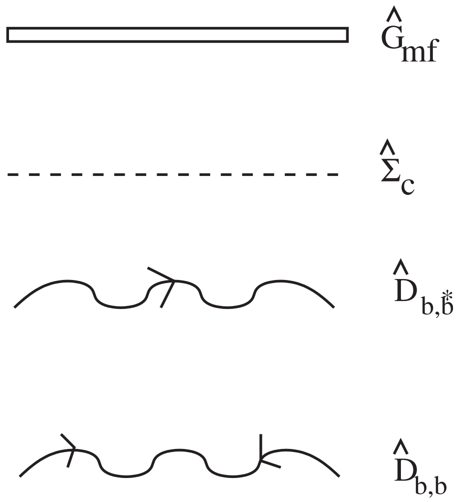

FIG. 1. Diagrams representing the mean-field Green's function $\hat{G}_{mf}$, the self-energy due to coupling to leads $\hat{\Sigma}_c$, and the bosonic propagator $\hat{D}$. Each has a 2×2 Keldysh structure.

$$\tilde{G}_{mf}=G_{mf}+\lambda_q G_{mf}\tau_x G_{mf}=G_{mf}+\lambda_q\delta G_{mf}, \tag{59}$$

where we define

$$\delta G_{mf}=G_{mf}\tau_x G_{mf}. \tag{60}$$

Expanding Eq. (57) to quadratic order in the fluctuating fields $b_{q,cl}$ we get

$$\begin{aligned}S_K={}&-iN\mathrm{Tr}\ln\tilde{G}^{-1}_{mf}(\lambda_q)+iNb_{sp}\mathrm{Tr}\{\tilde{G}_{mf}(\lambda_q)[\Sigma_c(b^*_{cl}\tau_0+b^*_q\tau_x)\\&+(b_{cl}\tau_0+b_q\tau_x)\Sigma_c]\}+\frac{iNb^2_{sp}}{2}\mathrm{Tr}[\{\tilde{G}_{mf}(\lambda_q)[\Sigma_c(b^*_{cl}\tau_0+b^*_q\tau_x)\\&+(b_{cl}\tau_0+b_q\tau_x)\Sigma_c]\}^2]+iN\mathrm{Tr}[\tilde{G}_{mf}(\lambda_q)(b_{cl}\tau_0+b_q\tau_x)\\&\times\Sigma_c(b^*_{cl}\tau_0+b^*_q\tau_x)]+2(b^*_{cl}\quad b^*_q)\begin{pmatrix}-\lambda_q & i\partial_t-\lambda_{cl}\\ i\partial_t-\lambda_{cl} & -\lambda_q\end{pmatrix}\\&\times\begin{pmatrix}b_{cl}\\ b_q\end{pmatrix}+2\lambda_q\left(1-\frac{N}{2}\right)-2\lambda_q b^2_{sp}-2\lambda_q b_{sp}(b_{cl}+b^*_{cl})\\&-2\lambda_{cl}b_{sp}(b_q+b^*_q).\end{aligned} \tag{61}$$

Collecting all terms up to quadratic order in the bosonic fields, we rewrite the action as below,

$$\begin{aligned}S_K={}&-iN\,\mathrm{Tr}\ln\tilde{G}^{-1}_{mf}(\lambda_q)+iNb_{sp}\mathrm{Tr}\{\tilde{G}_{mf}(\lambda_q)[\Sigma_c(b^*_{cl}\tau_0+b^*_q\tau_x)\\&+(b_{cl}\tau_0+b_q\tau_x)\Sigma_c]\}+2(b^*_{cl}\quad b^*_q)\left\{\begin{pmatrix}-\lambda_q & i\partial_t-\lambda_{cl}\\ i\partial_t-\lambda_{cl} & -\lambda_q\end{pmatrix}\right.\\&\left.-\Pi-\delta\Pi^{(1)}-\lambda_q[\delta\Pi_q+\delta\Pi^{(1)}_q]\right\}\begin{pmatrix}b_{cl}\\ b_q\end{pmatrix}+2(b^*_{cl}\quad b^*_q)\\&\times[-\delta\Pi^{(2)}-\lambda_q\delta\Pi^{(2)}_q]\begin{pmatrix}b^*_{cl}\\ b^*_q\end{pmatrix}\\&+2(b_{cl}\quad b_q)[-\delta\Pi^{(2)}-\lambda_q\delta\Pi^{(2)}_q]\begin{pmatrix}b_{cl}\\ b_q\end{pmatrix}\\&+2\lambda_q\left(1-\frac{N}{2}\right)-2\lambda_q b^2_{sp}-2\lambda_q b_{sp}(b_{cl}+b^*_{cl})\\&-2\lambda_{cl}b_{sp}(b_q+b^*_q).\end{aligned} \tag{62}$$

The above shows that the bosons due to their interaction with fermions acquire the self-energies $\Pi=\left(\begin{smallmatrix}0 & \Pi^A\\ \Pi^R & \Pi^K\end{smallmatrix}\right)$, and $\delta\Pi^{(1,2)}=\left(\begin{smallmatrix}0 & \delta\Pi^{A(1,2)}\\ \delta\Pi^{R(1,2)} & \delta\Pi^{K(1,2)}\end{smallmatrix}\right)$. The diagrams corresponding to $\Pi,\delta\Pi^{(1,2)}$ are shown in Fig. 2 (where the propagators are defined in Fig. 1). The bosonic self-energy Π is

$$\Pi(t,t')=\frac{-iN}{2}\begin{pmatrix}\mathrm{Tr}'[\tau_0 G_{mf}(t,t')\tau_0\Sigma_c(t',t)] & \mathrm{Tr}'[\tau_0 G_{mf}(t,t')\tau_x\Sigma_c(t',t)]\\ \mathrm{Tr}'[\tau_x G_{mf}(t,t')\tau_0\Sigma_c(t',t)] & \mathrm{Tr}'[\tau_x G_{mf}(t,t')\tau_x\Sigma_c(t',t)]\end{pmatrix}, \tag{63}$$

where Tr' implies trace over only the Keldysh indices. Note that from causality the upper left term in Eq. (63) is zero. Explicit expressions for Π are given in Appendix A. The other self-energies are

$$\begin{aligned}\delta\Pi^{A(1)}(t,t')={}&\frac{-iNb^2_{sp}}{4}[(G^K_{mf})_{t,t'}(\Sigma^R_c G^R_{mf}\Sigma^R_c)_{t',t}\\&+(G^A_{mf})_{t,t'}(\Sigma^R_c G^R_{mf}\Sigma^K_c)_{t',t}\\&+(G^A_{mf})_{t,t'}(\Sigma^R_c G^K_{mf}\Sigma^A_c)_{t',t}\\&+(G^A_{mf})_{t,t'}(\Sigma^K_c G^A_{mf}\Sigma^A_c)_{t',t}],\end{aligned} \tag{64}$$

$$\begin{aligned}\delta\Pi^{R(1)}(t,t')={}&\frac{-iNb^2_{sp}}{4}[(G^R_{mf})_{t,t'}(\Sigma^R_c G^R_{mf}\Sigma^K_c)_{t',t}\\&+(G^R_{mf})_{t,t'}(\Sigma^R_c G^K_{mf}\Sigma^A_c)_{t',t}\\&+(G^R_{mf})_{t,t'}(\Sigma^K_c G^A_{mf}\Sigma^A_c)_{t',t}\\&+(G^K_{mf})_{t,t'}(\Sigma^A_c G^A_{mf}\Sigma^A_c)_{t',t}],\end{aligned} \tag{65}$$

$$\begin{aligned}\delta\Pi^{K(1)}(t,t')={}&\frac{-iNb^2_{sp}}{4}\{(G^K_{mf})_{t,t'}(\Sigma^R_c G^R_{mf}\Sigma^K_c)_{t',t}\\&+(G^K_{mf})_{t,t'}(\Sigma^R_c G^K_{mf}\Sigma^A_c)_{t',t}\\&+(G^K_{mf})_{t,t'}(\Sigma^K_c G^A_{mf}\Sigma^A_c)_{t',t}\end{aligned}$$

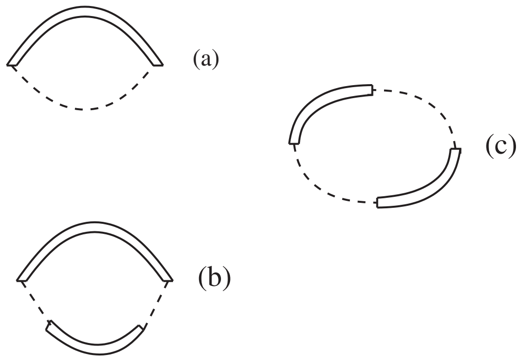

FIG. 2. The bosonic self-energies corresponding to (a) Π, (b) $\delta\Pi^{(1)}$, and (c). $\delta\Pi^{(2)}$ in text.

$$+(G^A_{\rm mf})_{t,t'}(\Sigma^R_c G^R_{\rm mf}\Sigma^R_c)_{t',t} + (G^R_{\rm mf})_{t,t'}(\Sigma^A_c G^A_{\rm mf}\Sigma^A_c)_{t',t}\}, \tag{66}$$

which are of $\mathcal{O}(1/N^2)$ and therefore will be dropped. The anomalous boson self-energies are the following:

$$\delta\Pi^{R(2)} = \frac{-iNb^2_{\rm sp}}{4}[(G^R_{\rm mf}\Sigma^R_c)_{t,t'}(G^R_{\rm mf}\Sigma^K_c)_{t't} + (G^R_{\rm mf}\Sigma^R_c)_{t,t'}(G^K_{\rm mf}\Sigma^A_c)_{t't} + (G^R_{\rm mf}\Sigma^K_c)_{t,t'}(G^A_{\rm mf}\Sigma^A_c)_{t't} + (G^K_{\rm mf}\Sigma^A_c)_{t,t'}(G^A_{\rm mf}\Sigma^A_c)_{t't}], \tag{67}$$

$$\delta\Pi^{A(2)} = \frac{-iNb^2_{\rm sp}}{4}[(G^A_{\rm mf}\Sigma^A_c)_{t,t'}(G^R_{\rm mf}\Sigma^K_c)_{t't} + (G^R_{\rm mf}\Sigma^K_c)_{t,t'}(G^R_{\rm mf}\Sigma^R_c)_{t't} + (G^A_{\rm mf}\Sigma^A_c)_{t,t'}(G^K_{\rm mf}\Sigma^A_c)_{t't} + (G^K_{\rm mf}\Sigma^A_c)_{t,t'}(G^R_{\rm mf}\Sigma^R_c)_{t't}], \tag{68}$$

$$\delta\Pi^{K(2)} = \frac{-iNb^2_{\rm sp}}{4}\{(G^R_{\rm mf}\Sigma^K_c)_{t,t'}(G^R_{\rm mf}\Sigma^K_c)_{t't} + (G^R_{\rm mf}\Sigma^K_c)_{t,t'}(G^K_{\rm mf}\Sigma^A_c)_{t't} + (G^K_{\rm mf}\Sigma^A_c)_{t,t'}(G^R_{\rm mf}\Sigma^K_c)_{t't} + (G^K_{\rm mf}\Sigma^A_c)_{t,t'}(G^K_{\rm mf}\Sigma^A_c)_{t't} + (G^A_{\rm mf}\Sigma^A_c)_{t,t'}(G^R_{\rm mf}\Sigma^R_c)_{t't} + (G^R_{\rm mf}\Sigma^R_c)_{t,t'}(G^A_{\rm mf}\Sigma^A_c)_{t't}\} \tag{69}$$

and are at least of $\mathcal{O}(1/N)$. These will therefore not play a role in the $\mathcal{O}(\frac{1}{N})$ corrections to the saddle-point equations, but will be important later, when we evaluate the conductance. The self-energies $\delta\Pi^{(2)}_q$ is also of $\mathcal{O}(1/N)$ and will be dropped from further consideration.

Other self-energies needed for computing corrections to the saddle-point equations are $\delta\Pi_q$ and $\delta\Pi^1_q$. We find

$$\delta\Pi_q(t,t') = \begin{pmatrix} \delta\Pi^z_q & \delta\Pi^A_q \\ \delta\Pi^R_q & \delta\Pi^K_q \end{pmatrix} \tag{70}$$

$$= \frac{-iN}{2}\begin{pmatrix} \mathrm{Tr}'[\tau_0\delta G_{\rm mf}(t,t')\tau_0\Sigma_c(t',t)] & \mathrm{Tr}'[\tau_0\delta G_{\rm mf}(t,t')\tau_x\Sigma_c(t',t)] \\ \mathrm{Tr}'[\tau_x\delta G_{\rm mf}(t,t')\tau_0\Sigma_c(t',t)] & \mathrm{Tr}'[\tau_x\delta G_{\rm mf}(t,t')\tau_x\Sigma_c(t',t)] \end{pmatrix} \tag{71}$$

with $\delta G_{\rm mf}$ defined in Eq. (60). Whereas $\delta\Pi^{(1)}_q$ is [retaining terms up to $\mathcal{O}(1/N)$]

$$\delta\Pi^{(1)}_q(t,t') = \begin{pmatrix} \delta\Pi^{z(1)}_q = \mathcal{O}(1/N^2) & \delta\Pi^{A(1)}_q \\ \delta\Pi^{R(1)}_q & \delta\Pi^{K(1)}_q = \mathcal{O}(1/N^2) \end{pmatrix}, \tag{72}$$

where

$$\delta\Pi^{R(1)}_q = \frac{-iNb^2_{\rm sp}}{2}[G^R_{\rm mf}(t,t')(\Sigma^R_c\delta G^K_{\rm mf}\Sigma^A_c)_{t',t} + G^R_{\rm mf}(t,t') \times(\Sigma^K_c\delta G^z_{\rm mf}\Sigma^K_c)_{t',t} + G^A_{\rm mf}(t,t')(\Sigma^A_c\delta G^z_{\rm mf}\Sigma^R_c)_{t',t}], \tag{73}$$

and

$$\delta\Pi^{A(1)}_q = \frac{-iNb^2_{\rm sp}}{2}[G^A_{\rm mf}(t,t')(\Sigma^R_c\delta G^K_{\rm mf}\Sigma^A_c)_{t',t} + G^A_{\rm mf}(t,t') \times(\Sigma^K_c\delta G^z_{\rm mf}\Sigma^K_c)_{t',t} + G^R_{\rm mf}(t,t')(\Sigma^A_c\delta G^z_{\rm mf}\Sigma^R_c)_{t',t}]. \tag{74}$$

We now integrate out the bosonic fields in the action Eq. (62). The $\mathcal{O}(\lambda^0_q)$ term cancels the last term because of the saddle-point condition Eq. (31), whereas the $\mathcal{O}(\lambda^1_q)$ term is also first order in the fluctuating bosonic fields $b_{q,\rm cl}$. Thus, on integrating out $b_{q,\rm cl}$, this term gives a term in the Keldysh action which is λ^2_q and therefore does not affect the classical saddle-point solutions. Following these steps we obtain

$$S_K = - iN \operatorname{Tr} \ln \tilde{G}_{\mathrm{mf}}^{-1}(\lambda_q) + i \operatorname{Tr} \ln[D_0^{-1} - \Pi - \lambda_q \tau_0 - \lambda_q(\delta\Pi_q + \delta\Pi_q^{(1)})] + 2\lambda_q\left(1 - \frac{N}{2}\right) - 2\lambda_q b_{\mathrm{sp}}^2 + \mathcal{O}(\lambda_q^2) + \mathcal{O}(1/N^2), \quad (75)$$

where $i\langle b_a b_b^* \rangle = \frac{1}{2}D_0$ is the bare bosonic propagator.

Now we may differentiate Eq. (75) with respect to λ_q and set all quantum fields to zero in the resultant expression to obtain

$$2\left(1 - \frac{N}{2}\right) - 2b_{\mathrm{sp}}^2 + iN \operatorname{Tr}[G_{\mathrm{mf}}^K] - i \operatorname{Tr}[D_{b,b^*} + D_{b,b^*}(\delta\Pi_q + \delta\Pi_q^{(1)})] = 0. \quad (76)$$

Up to $\mathcal{O}(1/N)$ only a subset of terms in Eq. (76) need to be kept. Collecting these,

$$2\left(1 - \frac{N}{2}\right) - 2b_{\mathrm{sp}}^2 + iN\operatorname{Tr}[G_{\mathrm{mf}}^K] - i\operatorname{Tr}D_{b,b^*}^K = \frac{N}{2}\int \frac{d\epsilon}{2\pi}\int \frac{d\omega}{2\pi}[D_{b,b^*}^A(\epsilon)G_{\mathrm{mf}}^A(\epsilon+\omega)G_{\mathrm{mf}}^K(\epsilon+\omega) + D_{b,b^*}^R(\epsilon)G_{\mathrm{mf}}^K(\epsilon+\omega)G_{\mathrm{mf}}^R(\epsilon+\omega) + D_{b,b^*}^K(\epsilon)G_{\mathrm{mf}}^A(\epsilon+\omega)G_{\mathrm{mf}}^R(\epsilon+\omega)]\Sigma^K(\omega). \quad (77)$$

Substituting for the fermionic and bosonic Green's functions [Eqs. (24), (25), (33), (A12), (A14), and (A15)], we get

$$1 - b_{\mathrm{sp}}^2 - \frac{N\tilde{\Gamma}}{\pi\epsilon_F}\sum_{a=L,R}\frac{\Gamma_a/\Gamma}{1-\mu_a/\epsilon_F} = \frac{N\Gamma\tilde{\Gamma}}{\pi^2 T_A^2}\sum_{a,b=L,R}\frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\dfrac{1}{1-\dfrac{\mu_a}{T_A}} - \dfrac{1}{1+x-\dfrac{\mu_b}{T_A}}\right)}{\left(x + m\sum_{i=L,R}\dfrac{\Gamma_i}{\Gamma}\ln\left|1+\dfrac{x}{1-\mu_i/T_A}\right|\right)^2} \times\left(1 + m\sum_i \frac{\Gamma_i}{\Gamma}\frac{1}{1+x-\mu_i/T_A}\right) + \frac{N\Gamma\tilde{\Gamma}}{\pi^2 T_A^2}\sum_{a,b=L,R}\frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\dfrac{1}{\left(1-\dfrac{\mu_a}{T_A}\right)^2} - \dfrac{1}{\left(1+x-\dfrac{\mu_b}{T_A}\right)^2}\right)}{x + m\sum_i\dfrac{\Gamma_i}{\Gamma}\ln\left|1+\dfrac{x}{1-\mu_i/T_A}\right|}. \quad (78)$$

We use the identity

$$1 + m\sum_i \frac{\Gamma_i/\Gamma}{1+x-\mu_i/T_A} = \sum_i \frac{\Gamma_i}{\Gamma}\left[1 + \frac{m}{1-\mu_i/T_A} - \frac{m}{1-\dfrac{\mu_i}{T_A}}\left(\frac{x}{1+x-\mu_i/T_A}\right)\right] = 1 + m_V - mx\sum_i\left(\frac{\Gamma_i/\Gamma}{(1+x-\mu_i/T_A)\left(1-\dfrac{\mu_i}{T_A}\right)}\right), \quad (79)$$

and for convenience introduce the following short-hand:

$$L_p^{a,b} = \frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\dfrac{1}{\left(1-\dfrac{\mu_a}{T_A}\right)^p} - \dfrac{1}{\left(1+x-\dfrac{\mu_b}{T_A}\right)^p}\right)}{x + m\sum_i\dfrac{\Gamma_i}{\Gamma}\ln\left|1+\dfrac{x}{1-\mu_i/T_A}\right|}, \quad (80)$$

$$k^{a,b} = \frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\dfrac{x+(\mu_a-\mu_b)/T_A}{(1-\mu_a/T_A)(1+x-\mu_b/T_A)}\right)x\sum_i\dfrac{\Gamma_i/\Gamma}{(1+x-\mu_i/T_A)(1-\mu_i/T_A)}}{\left(x + m\sum_i\dfrac{\Gamma_i}{\Gamma}\ln\left|1+\dfrac{x}{1-\mu_i/T_A}\right|\right)^2}, \quad (81)$$

$$I^{a,b} = \frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\frac{x+(\mu_a-\mu_b)/T_A}{(1-\mu_a/T_A)(1+x-\mu_b/T_A)}\right)}{\left(x+m\sum_i \frac{\Gamma_i}{\Gamma}\ln\left|1+\frac{x}{1-\mu_i/T_A}\right|\right)^2}. \tag{82}$$

Note that the functions $I^{a,b}$ are infrared divergent. In equilibrium ($\mu_L=\mu_R=0$) these have a logarithmic divergence, while out of equilibrium the $I^{a,b}$ have a more severe $1/x$ divergence. As shown in equilibrium by Read *et al.*,[21,22] in the computation of physical observables the $I^{a,b}$ appear in such a way as to exactly cancel the divergences. For the out-of-equilibrium calculation as well we find an exact cancellation of divergences, so that all physical observables are well defined.

In terms of the above symbols, Eq. (78) becomes

$$1-b_{\rm sp}^2-\frac{N\tilde{\Gamma}}{\pi\epsilon_F}\sum_{a=L,R}\frac{\Gamma_a/\Gamma}{1-\mu_a/\epsilon_F} = \frac{N\Gamma\tilde{\Gamma}}{\pi^2T_A^2}\sum_{a,b=L,R}[-mk^{a,b}+(1+m_V)I^{a,b}+L_2^{a,b}]. \tag{83}$$

The left-hand side of the above equation can be further arranged as follows by writing $\epsilon_F=T_A+\beta/N$:

$$\frac{N\tilde{\Gamma}}{\pi\epsilon_F}\sum_a\frac{\Gamma_a/\Gamma}{1-\mu_a/\epsilon_F} = \frac{N\tilde{\Gamma}}{\pi T_A}\sum_a\frac{\Gamma_a/\Gamma}{1-\mu_a/T_A} - \frac{\beta}{NT_A}\frac{N\tilde{\Gamma}}{\pi T_A}\sum_a\frac{\Gamma_a/\Gamma}{(1-\mu_a/T_A)^2}. \tag{84}$$

The $1/N$ correction to ϵ_F is carried out in the next subsection. Using the result for β derived there [Eq. (101)], the above equation is rewritten as

$$\frac{N\tilde{\Gamma}}{\pi\epsilon_F}\sum_a\frac{\Gamma_a/\Gamma}{1-\mu_a/\epsilon_F} = \frac{N\tilde{\Gamma}}{\pi T_A}\sum_a\frac{\Gamma_a/\Gamma}{1-\mu_a/T_A} - \left(\frac{m^2}{N(1+m_V)}\sum_{a,b}L_1^{a,b}\right)\frac{N\tilde{\Gamma}}{\pi T_A}\times\sum_a\frac{\Gamma_a/\Gamma}{(1-\mu_a/T_A)^2}. \tag{85}$$

Substituting Eq. (85) into Eq. (83) we obtain the following expression for $\tilde{\Gamma}$ up to $\mathcal{O}(1/N)$:

$$\frac{\tilde{\Gamma}}{\Gamma} = 1-\frac{m_V}{1+m_V}\left[1+\frac{1}{N}\sum_{a,b}\left\{\frac{m^2/m_V}{1+m_V}L_2^{a,b}-\frac{m^3/m_V}{1+m_V}k^{a,b} - \frac{m^3/m_V}{(1+m_V)^2}L_1^{a,b}\left[\sum_i\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^2}\right]\right\}\right]-\frac{1}{N}\frac{m^2}{1+m_V}\sum_{a,b}I^{a,b}. \tag{86}$$

It is convenient to introduce the following simplified notation:

$$k=\sum_{a,b}k^{a,b}, \tag{87}$$

$$I=\sum_{a,b}I^{a,b}, \tag{88}$$

$$L_p=\sum_{a,b}L_p^{a,b}, \tag{89}$$

$$S_{p=1,2,3}=\sum_i\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^p}. \tag{90}$$

Then,

$$\frac{\tilde{\Gamma}}{\Gamma} = 1-\frac{m_V}{1+m_V}\left[1+\frac{1}{N}\left(\frac{m^2/m_V}{1+m_V}L_2-\frac{m^3/m_V}{1+m_V}k - \frac{m^3/m_V}{(1+m_V)^2}L_1S_2\right)\right]-\frac{1}{N}\frac{m^2}{1+m_V}I. \tag{91}$$

Note that in equilibrium when $\mu_L=\mu_R=0$, Eq. (89) becomes

$$L_p^{\rm eq}=\int_0^{D/T_A}dx\frac{\left(1-\frac{1}{(1+x)^p}\right)}{x+m\ln(1+x)}, \tag{92}$$

which is an expression that will appear later in the voltage expansion for physical observables.

Thus the main result of this subsection is Eq. (91) which is the $1/N$ correction to the level broadening.

B. Saddle-point equation for b

In order to derive the $1/N$ corrections to the saddle-point Eq. (30), we set $\lambda_q=0$ in Eq. (6), write $b_{\rm cl}=b_{\rm sp}+b_{\rm cl}, b_{\rm cl}^*=b_{\rm sp}+b_{\rm cl}^*$ and $b_q=\bar{b}_q+b_q, b_q^*=\bar{b}_q+b_q^*$, and expand to quadratic order in the fluctuating fields $b_{q,\rm cl}$. Following this as before, we integrate out the fermions and the bosons and obtain an action $S_K(\bar{b}_q)$. To obtain the classical saddle point, we need

$\frac{\delta S_K(\bar{b}_q)}{\delta \bar{b}_q}|_{\bar{b}_q=0}$. We will now follow the above steps.

First we integrate out the electrons to obtain

$$\begin{aligned}S_K = & -iN\mathrm{Tr}\,\ln[G_{\mathrm{mf}}^{-1} - b_{\mathrm{sp}}\tau_0\Sigma_c\bar{b}_q\tau_x - \bar{b}_q\tau_x\Sigma_c b_{\mathrm{sp}}\tau_0 - \mathcal{O}(\bar{b}_q^2)\\ & - (b_{\mathrm{sp}}\tau_0 + \bar{b}_q\tau_x)\Sigma_c(b_{\mathrm{cl}}^*\tau_0 + b_q^*\tau_x) - (b_{\mathrm{cl}}\tau_0 + b_q\tau_x)\\ & \times\Sigma_c(b_{\mathrm{sp}}\tau_0 + \bar{b}_q\tau_x) - (b_{\mathrm{cl}}\tau_0 + b_q\tau_x)\Sigma_c(b_{\mathrm{cl}}^*\tau_0 + b_q^*\tau_x)]\\ & + 2(b_{\mathrm{cl}}^* \quad b_q^*)\begin{pmatrix} 0 & i\partial_t - \lambda_{\mathrm{cl}} \\ i\partial_t - \lambda_{\mathrm{cl}} & 0\end{pmatrix}\begin{pmatrix} b_{\mathrm{cl}} \\ b_q\end{pmatrix} - 4\lambda_{\mathrm{cl}}b_{\mathrm{sp}}\bar{b}_q\\ & - 2\lambda_{\mathrm{cl}}b_{\mathrm{sp}}(b_q + b_q^*) - 2\lambda_{\mathrm{cl}}\bar{b}_q(b_{\mathrm{cl}} + b_{\mathrm{cl}}^*).\end{aligned} \tag{93}$$

Expanding the above to quadratic order in the fluctuating fields we get

$$\begin{aligned}S_K = & iNb_{\mathrm{sp}}\bar{b}_q\mathrm{Tr}[G_{\mathrm{mf}}\{\tau_x\Sigma_c + \Sigma_c\tau_x\}] + iNb_{\mathrm{sp}}^2\bar{b}_q(b_q^*\mathrm{Tr}[G_{\mathrm{mf}}^R\Sigma_c^K\\ & + G_{\mathrm{mf}}^K\Sigma_c^A] + b_q[G_{\mathrm{mf}}^K\Sigma_c^R + G_{\mathrm{mf}}^A\Sigma_c^K]) + iN\mathrm{Tr}[(b_{\mathrm{sp}}b_q^* + b_{\mathrm{cl}}\bar{b}_q)\\ & \times(G_{\mathrm{mf}}^R\Sigma_c^K + G_{\mathrm{mf}}^K\Sigma_c^A) + (b_q b_{\mathrm{sp}} + \bar{b}_q b_{\mathrm{cl}}^*)(G_{\mathrm{mf}}^K\Sigma_c^R + G_{\mathrm{mf}}^A\Sigma_c^K)]\\ & + iN\bar{b}_q(b_q + b_q^*)\mathrm{Tr}[G_{\mathrm{mf}}^R\Sigma_c^A + G_{\mathrm{mf}}^A\Sigma_c^R + G_{\mathrm{mf}}^K\Sigma_c^K]\\ & + 2(b_{\mathrm{cl}}^* \quad b_q^*)[D_0^{-1} - \Pi - \delta\Pi^{(1)} - b_{\mathrm{sp}}\bar{b}_q(\delta\Pi_q' + \delta\Pi_q'^{(1)})]\\ & \times\begin{pmatrix} b_{\mathrm{cl}} \\ b_q\end{pmatrix} + 2(b_{\mathrm{cl}}^* \quad b_q^*)[-\delta\Pi^{(2)} - b_{\mathrm{sp}}\bar{b}_q\delta\Pi_q'^{(2)}]\begin{pmatrix} b_{\mathrm{cl}}^* \\ b_q^*\end{pmatrix}\\ & + 2(b_{\mathrm{cl}} \quad b_q)[-\delta\Pi^{(2)} - b_{\mathrm{sp}}\bar{b}_q\delta\Pi_q'^{(2)}]\begin{pmatrix} b_{\mathrm{cl}} \\ b_q\end{pmatrix} - 4\lambda_{\mathrm{cl}}b_{\mathrm{sp}}\bar{b}_q\\ & - 2\lambda_{\mathrm{cl}}b_{\mathrm{sp}}(b_q + b_q^*) - 2\lambda_{\mathrm{cl}}\bar{b}_q(b_{\mathrm{cl}} + b_{\mathrm{cl}}^*)\end{aligned} \tag{94}$$

with Π defined in Eq. (63), and the components of $\delta\Pi^1$ defined in Eqs. (64)–(66), and those of $\delta\Pi^2$ defined in Eqs. (67)–(69). Moreover, the $\delta\Pi_q'$ are given by

$$\delta\Pi_q'(t,t') = \frac{-iN}{2}\begin{pmatrix}\mathrm{Tr}\{[G_{\mathrm{mf}}(\Sigma_c\tau_x + \tau_x\Sigma_c)G_{\mathrm{mf}}](t,t')\Sigma_c(t',t)\} & \mathrm{Tr}\{[G_{\mathrm{mf}}(\Sigma_c\tau_x + \tau_x\Sigma_c)G_{\mathrm{mf}}](t,t')\tau_x\Sigma_c(t',t)\} \\ \mathrm{Tr}\{\tau_x[G_{\mathrm{mf}}(\Sigma_c\tau_x + \tau_x\Sigma_c)G_{\mathrm{mf}}](t,t')\Sigma_c(t',t)\} & \mathrm{Tr}\{\tau_x[G_{\mathrm{mf}}(\Sigma_c\tau_x + \tau_x\Sigma_c)G_{\mathrm{mf}}](t,t')\tau_x\Sigma_c(t',t)\}\end{pmatrix} \tag{95}$$

and $\delta\Pi_q'^{(1)} = \mathcal{O}(\frac{1}{N^2})$ and therefore will not play a role in the subsequent discussion.

Integrating out the bosonic fields and keeping terms up to $\mathcal{O}(1/N, \bar{b}_q)$ we get

$$\begin{aligned}S_K = & iNb_{\mathrm{sp}}\bar{b}_q\mathrm{Tr}[G_{\mathrm{mf}}\{\tau_x\Sigma_c + \Sigma_c\tau_x\}] - 4\lambda_{\mathrm{cl}}b_{\mathrm{sp}}\bar{b}_q\\ & - ib_{\mathrm{sp}}\bar{b}_q\mathrm{Tr}[D\delta\Pi_q'].\end{aligned} \tag{96}$$

Substituting for $\delta\Pi_q'$, to $\mathcal{O}(1/N)$, the above becomes

$$\begin{aligned}S_K = & iNb_{\mathrm{sp}}\bar{b}_q\mathrm{Tr}[G_{\mathrm{mf}}\{\tau_x\Sigma_c + \Sigma_c\tau_x\}] - 4\lambda_{\mathrm{cl}}b_{\mathrm{sp}}\bar{b}_q\\ & - Nb_{\mathrm{sp}}\bar{b}_q\int\frac{d\epsilon}{2\pi}\int\frac{d\omega}{2\pi}D^R(\epsilon)G_{\mathrm{mf}}^R(\epsilon+\omega)\\ & \times\Sigma^K(\epsilon+\omega)G_{\mathrm{mf}}^R(\epsilon+\omega)\Sigma_c^K(\omega).\end{aligned} \tag{97}$$

Thus the saddle-point equation for ϵ_F (obtained from $\frac{\delta S_K}{\delta \bar{b}_q}=0$) reduces to

$$\begin{aligned}\epsilon_F - E_0 + \frac{N\Gamma}{\pi}\sum_a \ln\frac{|\mu_a - \epsilon_F|}{D} = \frac{N\Gamma^2}{\pi^2 T_A}\Bigg[&\int_{-D}^{0} d\omega\frac{\left[\sum_{a,b=L,R}\frac{\Gamma_a\Gamma_b}{\Gamma^2}\left(\frac{1}{1-\frac{\omega+\mu_a}{\epsilon_F}} - \frac{1}{1-\frac{\mu_b}{\epsilon_F}}\right)\right]}{\omega - \frac{N\Gamma}{\pi}\sum_{a=L,R}\frac{\Gamma_a}{\Gamma}\ln\left|1-\frac{\omega}{\epsilon_F-\mu_a}\right|}\\ & + \sum_{a\neq b}\int_0^{\mu_a-\mu_b} d\omega\frac{\left[\frac{\Gamma_a\Gamma_b}{\Gamma^2}\left(\frac{1}{1-\frac{\omega+\mu_b}{\epsilon_F}} - \frac{1}{1-\frac{\mu_a}{\epsilon_F}}\right)\right]}{\omega - \frac{N\Gamma}{\pi}\sum_{a=L,R}\frac{\Gamma_a}{\Gamma}\ln\left|1-\frac{\omega}{\epsilon_F-\mu_a}\right|}\Bigg].\end{aligned} \tag{98}$$

The above equation may be used to extract the $\mathcal{O}(1/N)$ correction to the saddle-point expression for the level-energy ϵ_F. Writing

$$\epsilon_F = T_A + \frac{\beta}{N}, \tag{99}$$

T_A is given by Eq. (35), whereas from Eq. (98), we get

$$\beta = \frac{m^2 T_A}{1+m_V}\sum_{a,b}\frac{\Gamma_a\Gamma_b}{\Gamma^2} \times\left[\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\frac{1}{1-\mu_a/T_A} - \frac{1}{1+x-\mu_b/T_A}\right)}{1+m\sum_i \frac{\Gamma_i}{\Gamma}\ln\left|1+\frac{x}{1-\mu_i/T_A}\right|}\right] \tag{100}$$

with m_V defined in Eq. (37).

Using Eq. (80) we may write

$$\beta = \frac{m^2 T_A}{1+m_V}\sum_{a,b} L_1^{a,b} = \frac{m^2 T_A}{1+m_V} L_1. \tag{101}$$

Thus, the two main results of this section is the $\mathcal{O}(1/N)$ corrections to the level-broadening (b_{sp}^2) and level-position ($E_0+\lambda$) which are given in Eqs. (91) and (101), respectively. These results will be used in subsequent sections for the evaluation of various observables to $\mathcal{O}(\frac{1}{N})$.

VI. EVALUATION OF n_F TO $\mathcal{O}(\frac{1}{N})$

In this section we will evaluate the local charge density n_F to $\mathcal{O}(1/N)$. n_F is given by

$$n_F = \sum_m \langle f_m^\dagger f_m\rangle = \frac{N}{2}[1 - iG_f^K]. \tag{102}$$

Thus, we need to evaluate G_f^K to $\mathcal{O}(1/N)$. For this we start by writing the Dyson equation for the fermionic Green's function correct to one loop,

$$G_f = G_{\text{mf}} + G_{\text{mf}}\Sigma_F G_{\text{mf}}, \tag{103}$$

where the second term in the above equation corresponds to the diagram in Fig. 3, and the Σ_F are defined in Eqs. (B1) and (B2). The Keldysh component of Eq. (103) gives

$$G_f^K = G_{\text{mf}}^K + G_{\text{mf}}^R\Sigma_F^R G_{\text{mf}}^K + G_{\text{mf}}^R\Sigma_F^K G_{\text{mf}}^A + G_{\text{mf}}^K\Sigma_F^A G_{\text{mf}}^A. \tag{104}$$

We rewrite

$$n_F = n_F^0 + n_F^a + n_F^b, \tag{105}$$

where

$$n_F^0 = \frac{N}{2}[1 - iG_{\text{mf}}^K] \tag{106}$$

$$= \frac{N\tilde{\Gamma}}{\pi\epsilon_F}\sum_{a=L,R}\frac{\Gamma_a/\Gamma}{1-\mu_a/\epsilon_F} \tag{107}$$

$$n_F^a = \frac{-iN}{2}\text{Tr}[G_{\text{mf}}^R\Sigma_F^R G_{\text{mf}}^K + G_{\text{mf}}^K\Sigma_F^A G_{\text{mf}}^A], \tag{108}$$

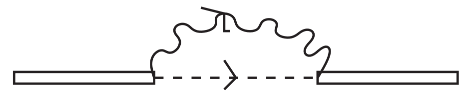

FIG. 3. Diagram contributing to the $1/N$ correction to n_F.

$$n_F^b = \frac{-iN}{2}\text{Tr}[G_{\text{mf}}^R\Sigma_F^K G_{\text{mf}}^A]. \tag{109}$$

We use Eqs. (91), (99), and (101) to correct $\tilde{\Gamma}/\Gamma, \epsilon_F$ to $\mathcal{O}(1/N)$ in Eq. (106) to obtain

$$n_F^0 = \frac{m_V}{1+m_V}\left[1 - \frac{1}{N}\frac{m^2}{1+m_V}\sum_{a,b}\left(L_2^{a,b} - \mu k^{a,b} + \frac{m/m_V}{1+m_V}\left(\sum_i\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^2}\right)L_1^{a,b} + (1+m_V)I^{a,b}\right)\right]. \tag{110}$$

Moreover, using Eqs. (B1) and (B2) one finds

$$n_F^a = \frac{1}{N}\frac{m^2}{1+m_V}\sum_{a,b} L_2^{a,b} \tag{111}$$

$$n_F^b = \frac{1}{N}\frac{m^2 m_V}{1+m_V}\sum_{a,b} I^{a,b} - \frac{1}{N}\frac{m^3}{1+m_V}\sum_{a,b} k^{a,b}. \tag{112}$$

Adding Eqs. (106), (111), and (112) gives

$$n_F = \frac{m_V}{1+m_V}\left[1 + \frac{1}{N}\sum_{a,b}\left(\frac{m^2/m_V}{1+m_V}L_2^{a,b} - \frac{m^3/m_V}{1+m_V}k^{a,b} - \frac{m^3/m_V}{(1+m_V)^2}L_1^{a,b}\sum_i\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^2}\right)\right]. \tag{113}$$

Comparing Eq. (113) with Eq. (86) we may write the expressions for $\frac{\tilde{\Gamma}}{\Gamma}$ in the following compact form:

$$\frac{\tilde{\Gamma}}{\Gamma} = 1 - n_F - \frac{1}{N}\frac{m^2}{1+m_V}\sum_{a,b} I^{a,b}. \tag{114}$$

Up to $\mathcal{O}(1/N)$, above may be rewritten as

$$\frac{\tilde{\Gamma}}{\Gamma} = 1 - n_F - (1-n_F)\frac{m^2}{N}\sum_{a,b} I^{a,b}. \tag{115}$$

The $I_{a,b}$ contain the divergent terms. If Λ is an infrared cutoff, and defining $V=|\mu_L-\mu_R|$, we find

$$\frac{\tilde{\Gamma}}{\Gamma} \simeq 1 - n_F - \frac{1}{N}\frac{m^2}{1+m_V}\frac{1}{(1+m_V)^2}\left(-\frac{\Gamma_L^2}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)^2}\ln\Lambda - \frac{\Gamma_R^2}{\Gamma^2}\frac{1}{(1-\mu_R/T_A)^2}\ln\Lambda - \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}\ln\frac{V}{T_A} + \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}\frac{V}{T_A}\int_0^{V/T_A}\frac{dx}{x^2}\right). \tag{116}$$

The above expression will be useful in the next section when we study the bosonic correlation function.

VII. BOSONIC CORRELATION FUNCTION: DECAY DUE TO CURRENT-INDUCED DECOHERENCE

The full bosonic correlation function (combining both saddle-point and fluctuation corrections) is

$$\bar{D}^K(t,t') = -i\langle\{b_{\rm sp}(t)+\delta b(t), b_{\rm sp}+\delta b^\dagger(t')\}\rangle$$
$$= -2i\frac{\tilde{\Gamma}}{\Gamma} + D^K_{b,b^*}(t,t'), \tag{117}$$

where $D^K_{b,b^*}=-i\langle\{\delta b(t),\delta b^\dagger(t')\}\rangle$, and its expression in frequency space is given in Eq. (A12). Using Eq. (A12)

$$D^K_{b,b^*}(t) = \int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-i\Omega t}D^K_{b,b^*}(\Omega)$$
$$= \frac{-i}{N}\frac{m^2}{1+m_V}\sum_{a,b=L,R}\frac{\Gamma_a\Gamma_b}{\Gamma^2}\gamma_{ab}(t), \tag{118}$$

where

$$\gamma_{ab}(t) = \int_{-\infty}^{\infty}d\Omega e^{-i\Omega t}\text{sgn}(\Omega+\mu_a-\mu_b) \times \frac{\dfrac{\Omega+\mu_a-\mu_b}{\left(1-\dfrac{\Omega+\mu_a}{T_A}\right)\left(1-\dfrac{\mu_b}{T_A}\right)}}{\left(\Omega-\dfrac{N\Gamma}{\pi}\sum_i\dfrac{\Gamma_i}{\Gamma}\ln\left|1-\dfrac{\Omega}{T_A-\mu_i}\right|\right)^2}, \tag{119}$$

where in the long-time limit,

$$\gamma_{aa}(t) = \frac{2}{\left(1-\dfrac{\mu_a}{T_A}\right)^2(1+m_V)^2}\int_0^\infty d\Omega\frac{1}{\Omega}\cos\Omega t$$
$$\simeq \frac{-2}{\left(1-\dfrac{\mu_a}{T_A}\right)^2(1+m_V)^2}\ln(\Lambda t T_A). \tag{120}$$

Λ is a cutoff introduced to take care of the infrared divergences. This term, as we shall show will be canceled by the corresponding infrared divergence from Eq. (91).

Similarly one finds (for $V=|\mu_L-\mu_R|$)

$$\gamma_{LR}+\gamma_{RL} = \frac{2}{\left(1-\dfrac{\mu_L}{T_A}\right)\left(1-\dfrac{\mu_R}{T_A}\right)(1+m_V)^2} \times\left[2\int_V^\infty\frac{d\Omega}{\Omega}\cos\Omega t + 2V\int_0^V\frac{d\Omega}{\Omega^2}\cos\Omega t\right]. \tag{121}$$

Therefore, the full bosonic correlation function is

$$\bar{D}^K(t) = \simeq -2i\left[\frac{\tilde{\Gamma}}{\Gamma}+\frac{1}{N}\frac{m^2}{1+m_V}\frac{1}{(1+m_V)^2}\times\left(-\frac{\Gamma_L^2}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)^2}\ln(\Lambda t T_A) - \frac{\Gamma_R^2}{\Gamma^2}\frac{1}{(1-\mu_R/T_A)^2}\ln(\Lambda t T_A) + \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}\int_V^\infty\frac{d\Omega}{\Omega}\cos\Omega t + \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}V\int_0^V\frac{d\Omega}{\Omega^2}\cos\Omega t\right)\right]. \tag{122}$$

Combining the above with expression for $\tilde{\Gamma}/\Gamma$ in Eq. (116), one finds that the infrared divergences cancel to give

$$\bar{D}^K(t) = -2i(1-n_F)\left[1-\frac{1}{N}\frac{m^2}{(1+m_V)^2}\times\left(\frac{\Gamma_L^2}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)^2}\ln(tT_A) + \frac{\Gamma_R^2}{\Gamma^2}\frac{1}{(1-\mu_R/T_A)^2}\ln(tT_A) - \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}\left(\ln\frac{V}{T_A} + \int_V^\infty\frac{d\Omega}{\Omega}\cos\Omega t\right) - \frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}\frac{V}{T_A}\times\int_0^{V/T_A}\frac{dx}{x^2}(\cos xT_A-1)\right)\right]. \tag{123}$$

Note that the above expression is correct to $\mathcal{O}(1/N)$. Therefore $1-n_F$ needs to be computed only to the saddle-point level for all terms except the first term in the square brackets.

For long times $Vt\gg 1$, Eq. (123) reduces to

$$\bar{D}^K(t) \simeq -2i(1-n_F)\left[1-\frac{c_L}{N}\ln tT_A-\frac{c_R}{N}\ln tT_A-\frac{c_{\rm dec}}{N}Vt\right], \tag{124}$$

where

$$c_{L,R} = \frac{m^2}{(1+m_V)^2}\frac{\Gamma_{L,R}^2}{\Gamma^2}\frac{1}{(1-\mu_{L,R}/T_A)^2} \tag{125}$$

$$c_{\rm dec} = \frac{m^2}{(1+m_V)^2}\frac{2\Gamma_L\Gamma_R}{\Gamma^2}\frac{1}{(1-\mu_L/T_A)(1-\mu_R/T_A)}. \tag{126}$$

If one were to compute the correlation function to higher orders in $\frac{1}{N}$, Eq. (124) signals the following behavior:

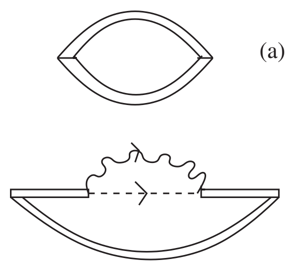

FIG. 4. Diagrams contributing to the susceptibility to $\mathcal{O}(1/N)$.

$$\bar{D}^K(t) \sim -2i(1-n_F)\exp\left[-\frac{1}{N}\sum_{i=L,R} c_i \ln tT_A - \frac{c_{\rm dec}}{N}Vt\right]. \tag{127}$$

Thus the slow power-law decay in time in equilibrium of the bosonic correlator is replaced by a rapid exponential decay at nonzero voltages whose origin is current-induced decoherence. Each of the exponents $c_{i,\rm dec}$ is consistent with what one might expect from nonequilibrium x-ray edge physics.[26]

The above decoherence rate appearing in the bosonic correlation function has consequences for physical observables such as the susceptibility and the conductance which we evaluate in subsequent sections.

VIII. EVALUATION OF SUSCEPTIBILITY TO $\mathcal{O}(\frac{1}{N})$

The spin-response function is given by

$$\chi^R(\Omega) = \frac{i}{2}\sum_m (g\mu_B m)^2 \int \frac{d\omega}{2\pi}[G_f^R(\omega+\Omega)G_f^K(\omega) + G_f^K(\omega+\Omega)G_f^A(\omega)]. \tag{128}$$

To $\mathcal{O}(1/N)$, from Eq. (128)

$$\chi^R(\Omega=0) = \chi_0^R + \chi_1^R + \chi_2^R, \tag{129}$$

where χ_0^R is the saddle-point expression for the susceptibility [diagram (a) in Fig. 4] with the level energy and broadening corrected to $\mathcal{O}(1/N)$, while $\chi_{1,2}^R$ arises due to corrections to the electronic Green's functions to one loop [diagram (b) in Fig. 4]. In particular,

$$\chi_1^R = i\sum_m (g\mu_B m)^2 \mathrm{Tr}[G_{\rm mf}^R \Sigma_f^R G_{\rm mf}^R G_{\rm mf}^K] \tag{130}$$

$$\chi_2^R = i\sum_m (g\mu_B m)^2 \mathrm{Tr}[\{G_{\rm mf}^R(\Sigma_f^R G_{\rm mf}^K + \Sigma_f^K G_{\rm mf}^A) + G_{\rm mf}^K \Sigma_f^A G_{\rm mf}^A\} G_{\rm mf}^R]. \tag{131}$$

In order to compute χ_0^R, we use Eq. (46) and correct for $\tilde{\Gamma}/\Gamma$ using Eq. (91), and correct for T_A using Eq. (101). We find

$$\chi_0^R = \chi_{\rm sp}\left[1 + \frac{1}{N}\left(-\frac{m^2}{1+m_V}L_2 + \frac{m^3}{1+m_V}k + \left[-2\frac{m^2}{1+m_V}\frac{S_3}{S_2^2} + \frac{m^3}{(1+m_V)^2}\right]L_1 S_2 - m^2 I\right)\right]. \tag{132}$$

Using Eq. (B1) it is straightforward to show

$$\chi_1^R = \left(\frac{g^2\mu_B^2}{3}J(J+1)\right)\frac{1}{T_A}\frac{4}{3}\frac{1}{N}\left(\frac{m^2}{1+m_V}\right)L_3 = \chi_{\rm sp}^R \frac{2}{3}\frac{1}{N}\frac{mL_3}{S_2}, \tag{133}$$

whereas

$$\chi_2^R = 2\chi_1^R + \frac{g^2\mu_B^2}{T_A}\frac{J(J+1)}{3}\frac{1}{N}\frac{m^3}{1+m_V}\sum_{ab}\frac{\Gamma_a\Gamma_b}{\Gamma^2} \times \int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \frac{\left(\frac{1}{1-\mu_a/T_A} - \frac{1}{1+x-\mu_b/T_A}\right)}{\left(x + m\sum_i \frac{\Gamma_i}{\Gamma}\ln\left|1 + \frac{x}{1-\mu_i/T_A}\right|\right)^2} \times \sum_i \frac{\Gamma_i/\Gamma}{(1+x-\mu_i/T_A)^2}. \tag{134}$$

Collecting all the terms together

$$\chi^R = \chi_{\rm sp}\left[1 + \frac{1}{N}\left(-\frac{m^2}{1+m_V}L_2 + \frac{m^3}{1+m_V}k + \frac{m^3}{(1+m_V)^2}L_1 S_2 - 2\frac{S_3}{S_2}\frac{m^2}{1+m_V}L_1 + 2mL_3 S_2 - m^2 I + F\right)\right], \tag{135}$$

where

$$F = \frac{m^2}{S}\sum_{a,b}\frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A} dx \times \frac{\left(\frac{1}{1-\mu_a/T_A} - \frac{1}{1+x-\mu_b/T_A}\right)}{\left(x + m\sum_i \frac{\Gamma_i}{\Gamma}\ln\left|1 + \frac{x}{1-\mu_i/T_A}\right|\right)^2} \times \sum_i \frac{\Gamma_i/\Gamma}{(1+x-\mu_i/T_A)^2}. \tag{136}$$

Now it can be shown that

$$\frac{m^3}{1+m_V}k - m^2 I + F = -\frac{m^2}{1+m_V}M, \tag{137}$$

where

$$M = \sum_{a,b} M^{a,b},$$

$$M^{a,b}=\frac{\Gamma_a\Gamma_b}{\Gamma^2}\int_{-(\mu_a-\mu_b)/T_A}^{D/T_A}dx \times\frac{\left(\frac{x+(\mu_a-\mu_b)/T_A}{(1-\mu_a/T_A)(1+x-\mu_b/T_A)}\right)}{\left(x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left|1+\frac{x}{1-\mu_i/T_A}\right|\right)^2}, \tag{138}$$

$$\sum_i\frac{\Gamma_i}{\Gamma}\left[\left(\frac{1+m_V}{S}\right)\frac{x^2+2x(1-\mu_i/T_A)}{(1+x-\mu_i/T_A)^2(1-\mu_i/T_A)^2} -\frac{mx}{(1+x-\mu_i/T_A)(1-\mu_i/T_A)}\right]. \tag{139}$$

Therefore the static spin susceptibility becomes

$$\chi^R=\chi_{\rm sp}\left[1+\frac{1}{N}\left(-\frac{m^2}{1+m_V}L_2+\frac{m^3}{(1+m_V)^2}L_1S_2 -2\frac{S_3}{S_2}\frac{m^2}{1+m_V}L_1+2m\frac{L_3}{S_2}-\frac{m^2}{1+m_V}M\right)\right] \tag{140}$$

with $\chi_{\rm sp}$ given in Eq. (48). Equation (140) may be expanded in powers of $\mu_{L,R}/T_A$. In particular in the Kondo limit ($n_F\to 1$ or $m_V\gg 1$), Eq. (140) is found to have the form

$$\chi_S^{n_F=1}=\frac{g^2\mu_B^2J(J+1)}{3T_A^0}\left[1+1.5\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\mu_L-\mu_R}{T_A^0}\right)^2\right] \times\left[1+\frac{m}{N}(-L_2^{\rm eq}-L_1^{\rm eq}+2L_3^{\rm eq}-mJ_0^{\rm eq}) +\frac{1}{N}\left(\sum_{i=L,R}\frac{\Gamma_i\mu_i}{\Gamma T_A^0}\right)C_{S1}+\frac{1}{N}\left(\sum_{i=L,R}\frac{\Gamma_i\mu_i^2}{\Gamma(T_A^0)^2}\right)C_{S2} +\frac{1}{N}\left(\sum_{i=L,R}\frac{\Gamma_i\mu_i}{\Gamma T_A^0}\right)^2(C_{S3}-C_{S1}) -\frac{4.5}{N}\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\mu_L-\mu_R}{T_A^0}\right)^2\right]. \tag{141}$$

The expressions for C_{Si} have been given in Appendix C, and may be evaluated numerically. The $L_p^{\rm eq}$ are defined in Eq. (92), and

$$J_0^{\rm eq}=\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^2}\frac{x^2}{(1+x)^3}. \tag{142}$$

Let us assume that the chemical potential of the left lead is $\mu_L=V/2$, and that for the right lead is $\mu_R=-V/2$. Let us define

$$C_{S0}=m(-L_2^{\rm eq}-L_1^{\rm eq}+2L_3^{\rm eq}-mJ_0^{\rm eq}). \tag{143}$$

We now rewrite Eq. (141) as follows:

$$\chi_S^{n_F=1}=\frac{g^2\mu_B^2J(J+1)}{3T_A^0}\left(1+\frac{C_{S0}}{N}\right)\left[1+1.5\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2 \times\left(1+\frac{2C_{S0}}{N}-\frac{2C_{S0}}{N}\right)+\frac{1}{N}\left(\frac{\Gamma_L-\Gamma_R}{\Gamma}\right)\left(\frac{V}{2T_A^0}\right)C_{S1} +\frac{1}{N}\left(\frac{V}{2T_A^0}\right)^2C_{S2}+\frac{1}{N}\left(\frac{\Gamma_L-\Gamma_R}{\Gamma}\right)^2\left(\frac{V}{2T_A^0}\right)^2(C_{S3}-C_{S1}) -\frac{4.5}{N}\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_A^0}\right)^2\right], \tag{144}$$

where terms higher order than $\frac{1}{N}\frac{V^2}{(T_A^0)^2}$ have been dropped. Defining the Kondo temperature to $\mathcal{O}(1/N)$ as[22]

$$T_K=T_A^0\left(1-\frac{C_{S0}}{N}\right), \tag{145}$$

Eq. (141) can be recast in the following universal form:

$$\chi_S^{n_F=1}=\frac{g^2\mu_B^2J(J+1)}{3T_K}\left[1+1.5\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_K}\right)^2+\frac{1}{N}\left(\frac{\Gamma_L-\Gamma_R}{\Gamma}\right) \times\left(\frac{V}{2T_K}\right)C_{S1}+\frac{1}{N}\left(\frac{V}{2T_K}\right)^2(C_{S2}+C_{S3}-C_{S1}) -\frac{1}{N}(4.5+3C_{S0}+C_{S3}-C_{S1})\frac{\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{V}{T_K}\right)^2\right]. \tag{146}$$

In the Kondo limit $m\gg 1$, the coefficients in the above equation take the following universal values: $C_{S1}\to 0.01$, $(C_{S2}+C_{S3}-C_{S1})\to -0.005$, and $(3C_{S0}+C_{S3}-C_{S1})\to -4.94$.

IX. EVALUATION OF THE SPECTRAL DENSITY AND CONDUCTANCE TO $\mathcal{O}(\frac{1}{N^2})$

The retarded Green's function, whose imaginary part gives the impurity spectral density is

$$G_{f,b}^R(t,t')=-iT\langle b_-^\dagger(t)f_-(t)f_-^\dagger(t')b_-(t')\rangle -i\langle f_+^\dagger(t')b_+(t')b_-^\dagger(t)f_-(t)\rangle \tag{147}$$

$$=i[G_{--}^f(t,t')D_{--}^{b,b^*}(t',t)-G_{-+}^f(t,t')D_{+-}^{b,b^*}(t',t)]. \tag{148}$$

We are only interested in evaluating the imaginary part of Eq. (148) which at leading order (saddle point level) is $\mathcal{O}(\frac{1}{N})$. There are five diagrams that contribute to the above expression to $\mathcal{O}(\frac{1}{N^2})$ which are shown in Fig. 5. For convenience we write

$$\mathrm{Im}[G_{f_m,b}^R]=T_a+T_b+T_c+T_d+T_e, \tag{149}$$

where T_i is the contribution from the ith diagram and corresponds to

$$T_a(\Omega)=\frac{-i\Gamma}{(T_A-\Omega)^2}\frac{1}{(1+m_V)^2}\left[1+\frac{2}{N}\left(-\frac{m^2}{1+m_V}L_2+\frac{m^3}{1+m_V}k +\frac{m^3}{(1+m_V)^2}L_1S-\frac{m^2}{1+m_V}\frac{L_1}{\left(1-\frac{\Omega}{T_A}\right)}-\mu^2I\right)\right], \tag{150}$$

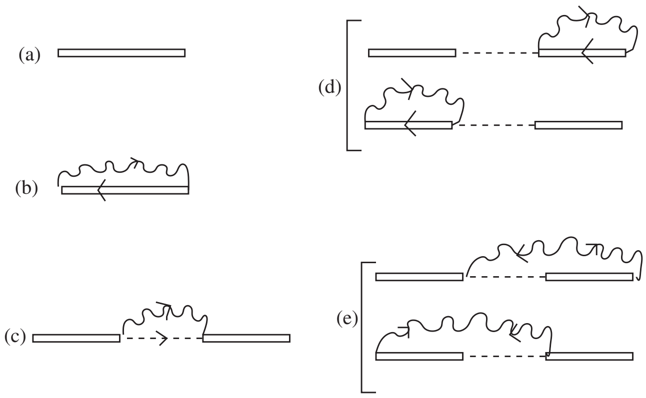

FIG. 5. Diagrams needed for the computation of the impurity spectral density to $\mathcal{O}(1/N^2)$ and hence the conductance to $\mathcal{O}(1/N^2)$.

$$T_b(\Omega) = \mathrm{Im}\left[\frac{i}{2}\int \frac{d\omega}{2\pi}\{G^K_{\mathrm{mf}}(\omega+\Omega)D^A_{b,b^*}(\omega) + G^R_{\mathrm{mf}}(\omega+\Omega)D^K_{b,b^*}(\omega)\}\right], \tag{151}$$

$$T_c(\Omega) = \frac{\tilde{\Gamma}}{\Gamma}\mathrm{Im}\left\{[G^R_{\mathrm{mf}}(\Omega)]^2\frac{i}{2}\int \frac{d\omega}{2\pi}[D^R_{b,b^*}(\omega+\Omega)\Sigma^K_c(-\omega) + D^K_{b,b^*}(\omega+\Omega)\Sigma^R_c(-\omega)]\right\}, \tag{152}$$

$$T_d(\Omega) = 2\frac{\tilde{\Gamma}}{\Gamma}\mathrm{Im}\left[G^R_{\mathrm{mf}}(\Omega)\Sigma^R_c(\Omega)\frac{i}{2}\int \frac{d\omega}{2\pi}\{G^K_{\mathrm{mf}}(\omega+\Omega)D^A_{b,b^*}(\omega) + G^R_{\mathrm{mf}}(\omega+\Omega)D^K_{b,b^*}(\omega)\}\right], \tag{153}$$

and

$$T_e(\Omega) = \frac{\tilde{\Gamma}}{\Gamma}\mathrm{Im}\left[G^R_{\mathrm{mf}}(\Omega)\frac{i}{2}\int \frac{d\omega}{2\pi}[D^R_{b,b}(\omega+\Omega)\{[\Sigma^R_c(-\omega) + \Sigma^A_c(-\omega)]G^K_{\mathrm{mf}}(-\omega) + \Sigma^K_c(-\omega)[G^R_{\mathrm{mf}}(-\omega) + G^A_{\mathrm{mf}}(-\omega)]\} + D^K_{b,b}(\omega+\Omega)\Sigma^R_c(-\omega)G^R_{\mathrm{mf}}(-\omega)]\right]. \tag{154}$$

The above terms have been evaluated in Appendix D.

We now present results for the conductance for the case of symmetric couplings to the leads ($\Gamma_L = \Gamma_R$) and $\mu_L = V/2, \mu_R = -V/2$. Defining

$$G_0 = \frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\pi}{N}\right)^2\left(\frac{m_0}{1+m_0}\right)^2 \tag{155}$$

and the functions

$$k^0 = \int_0^{D/T_A} dx \frac{\left(\frac{x}{1+x}\right)^2}{[x+m\ \ln(1+x)]^2}, \tag{156}$$

$$p^0 = \int_0^{D/T_A} dx \frac{\left(\frac{x}{1+x}\right)^2\frac{(x^2+3x+3)}{(1+x)^2}}{[x+m\ \ln(1+x)]^2}, \tag{157}$$

$$p^1 = \int_0^{D/T_A} dx \frac{\left(\frac{x}{1+x}\right)^2\frac{(3x^2+8x+6)}{(1+x)^2}}{[x+m\ \ln(1+x)]^2}, \tag{158}$$

$$t_1 = \frac{1}{2}\int_0^{D/T_A} \frac{dx}{\{[x+m\ \ln(1+x)]^2}\frac{x(x+2)}{(1+x)^2}\left[1-\frac{1}{(1+x)^2}\right], \tag{159}$$

$$t_2 = \int_0^{D/T_A} \frac{dx}{\{[x+m\ \ln(1+x)]^3}\frac{x(x+2)}{(1+x)^2}\frac{x^2}{(1+x)^2}, \tag{160}$$

$$t_3 = \frac{1}{2}\int_0^{D/T_A} \frac{dx}{\{[x+m\ \ln(1+x)]^2}\frac{x(x+2)}{(1+x)^2}\left(1-\frac{1}{1+x}\right), \tag{161}$$

we obtain the following expression for the conductance:

$$\frac{G(V)}{G_0} = 1 + \frac{2}{N}\frac{m_0}{1+m_0}\left[L^{\mathrm{eq}}_2 - m_0k^0 - \frac{m_0}{1+m_0}L^{\mathrm{eq}}_1\right] - \frac{3V^2}{4(T^0_A)^2}\frac{m_0}{1+m_0} + , \tag{162}$$

$$+ \frac{1}{N}\frac{V^2}{(T^0_A)^2}\left[-\frac{m_0}{1+m_0} + \frac{17}{2}\left(\frac{m_0}{1+m_0}\right)^2 - 6\left(\frac{m_0}{1+m_0}\right)^3 + \frac{5}{4}\left(\frac{m_0}{1+m_0}\right)^4\right] + , \tag{163}$$

$$+ \frac{1}{N}\frac{3V^2}{4(T^0_A)^2}\left[-m_0t_1 + m_0^2t_2 + \frac{m_0^2}{1+m_0}t_3\right], \tag{164}$$

$$+ \frac{1}{N}\frac{V^2}{2(T^0_A)^2}\left[\frac{m_0(m_0^2-7m_0-1/2)}{(1+m_0)^2}L^{\mathrm{eq}}_2 + \frac{3m_0^2(7m_0+2)}{2(1+m_0)^2}k^0 + \frac{3m_0^2(m_0^2+3m_0-1)}{(1+m_0)^3}L^{\mathrm{eq}}_1 + \frac{m_0(2m_0^2+m_0+2)}{(1+m_0)^2}L^{\mathrm{eq}}_3 - \frac{3m_0(2m_0-1)}{1+m_0}L^{\mathrm{eq}}_4 + \frac{3m_0^2(m_0-1)}{1+m_0}p^0 - m_0^2p^1\right]. \tag{165}$$

Note that in equilibrium, the above equation reduces to[22] $G = \frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}(\frac{\pi n_F}{N})^2$ as expected from the Friedel sum rule.

The expression for the conductance in the Kondo regime may be obtained by setting $m_0 \gg 1$ in the above equation. We find

$$G^{n_F=1}(V;\Gamma_L=\Gamma_R)=\frac{Ne^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2}\left(\frac{\pi}{N}\right)^2\left[1-\frac{3}{4}\left(\frac{V}{T_K}\right)^2+\frac{1}{2N}\left(\frac{V}{T_K}\right)^2(5.5+C_{G1})\right] \tag{166}$$

with T_K defined in Eq. (145), and the coefficient C_{G1} given in Eq. (C7). In the Kondo limit, $C_{G1}\to-2.77$.

X. CONCLUSIONS

In summary, we have presented results for the nonequilibrium infinite-U Anderson model using Keldysh functional integral methods. The approach has been to use $1/N$ as the small parameter in the theory which allows us to develop a systematic perturbation theory for the nonequilibrium problem. The results derived are valid for an applied voltage small as compared to the Kondo temperature when the effect of fluctuations are small. Physical quantities such as the impurity spin susceptibility and the conductance are calculated to $\mathcal{O}(\frac{1}{N}(\frac{V}{T_K})^2)$. The voltage expansions are found to show rich behavior by depending on different combinations of the couplings to the left and right leads such as $(\frac{\Gamma_L-\Gamma_R}{\Gamma})\frac{V}{T_K}, \frac{\Gamma_L^2+\Gamma_R^2}{\Gamma^2}(\frac{V}{T_K})^2, \frac{\Gamma_L\Gamma_R}{\Gamma^2}(\frac{V}{T_K})^2$. While terms of the first kind give rise to rectification-type behavior, i.e., $\chi_S(V)\neq\chi_S(-V)$ and $G(V)\neq G(-V)$, the last term is associated with current-induced decoherence as it arises due to inelastic processes that can occur in an energy window V. This term is also found to cause the bosonic correlation function to decay rapidly in time. The approach developed in this paper is rather general and therefore may be easily adaptable to a variety of out-of-equilibrium systems.

An interesting question that arises is to what extent the results obtained in this paper are also valid for $N=2$. It is known for equilibrium systems that a naive extrapolation of the results of large N to $N=2$ when compared with exact Bethe-Ansatz results not only gives incorrect numerical values of various quantities (such as the Wilson ratio and the zero-bias conductance) but also makes qualitatively incorrect predictions for the temperature dependence of observables. Precisely how the extrapolation goes wrong has been discussed in Appendix E. On the other hand, comparison with exact results[17] reveals that large-N works very well for $N\geq 4$. However, one of the results of this paper has been the observation that $G(-V)\neq G(V)$, $\chi_S(V)\neq\chi_S(-V)$ for unequal coupling to the two leads. This asymmetry is rather generic and will exist whenever the system is away from particle-hole symmetry and therefore should be observed for the nonequilibrium $N=2$ Anderson model away from the particle-hole symmetry point $E_0=-\frac{U}{2}$. However, for a small voltage expansion of the conductance for $N=2$ we do not expect the appearance of a linear in voltage term as found in Eq. (13). This is because the $N=2$ case has a maximal conductance per channel of e^2/h, and such a linear term would imply that the conductance can become larger than this value, which is unphysical. An asymmetry can very well appear at cubic order $(\frac{\Gamma_L-\Gamma_R}{\Gamma}(\frac{V}{T_K})^3)$ in the small voltage expansion of G. Note that for $N\gg 1$, the conductance per channel is a small number of $\mathcal{O}(1/N^2)$. Therefore for this case a linear in voltage term in the conductance does not violate unitarity.

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APPENDIX A: EVALUATION OF BOSONIC SELF-ENERGIES AND PROPAGATORS

In this section we evaluate explicit expressions for $\Pi^{R,A,K}$ defined in Eq. (63). In particular

$$\Pi^R(1,2)=\frac{-iN}{2}[G^R_{\rm mf}(1,2)\Sigma^K_c(2,1)+G^K_{\rm mf}(1,2)\Sigma^A_c(2,1)], \tag{A1}$$

$$\Pi^A(1,2)=\frac{-iN}{2}[G^A_{\rm mf}(1,2)\Sigma^K_c(2,1)+G^K_{\rm mf}(1,2)\Sigma^R_c(2,1)], \tag{A2}$$

$$\Pi^K(1,2)=\frac{-iN}{2}[G^R_{\rm mf}(1,2)\Sigma^A_c(2,1)+G^A_{\rm mf}(1,2)\Sigma^R_c(2,1)+G^K_{\rm mf}(1,2)\Sigma^K_c(2,1)]. \tag{A3}$$

The above may be easily evaluated. We obtain

$$\Pi_R(\Omega)=\Pi_R'(\Omega)+i\Pi_R''(\Omega), \tag{A4}$$

where

$$\Pi_R'(\Omega)=\frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\frac{\sqrt{(\Omega+\mu_L-E_0-\lambda_{\rm cl})^2+\tilde{\Gamma}^2}}{D}+\frac{\Gamma_R}{\Gamma}\ln\frac{\sqrt{(\Omega+\mu_R-E_0-\lambda_{\rm cl})^2+\tilde{\Gamma}^2}}{D}\right], \tag{A5}$$

$$\Pi_R''(\Omega)=\frac{-N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\left(\arctan\frac{\tilde{\Gamma}}{\mu_L-E_0-\lambda_{\rm cl}}-\arctan\frac{\tilde{\Gamma}}{\Omega+\mu_L-E_0-\lambda_{\rm cl}}\right)+\frac{\Gamma_R}{\Gamma}\left(\arctan\frac{\tilde{\Gamma}}{\mu_R-E_0-\lambda_{\rm cl}}-\arctan\frac{\tilde{\Gamma}}{\Omega+\mu_R-E_0-\lambda_{\rm cl}}\right)\right]. \tag{A6}$$

Defining $\lambda_{\rm cl}+E_0=\epsilon_F$, and to $\mathcal{O}(1/N)$, the above expressions simplify to

$$\Pi_R'(\Omega)=\frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\frac{|\Omega+\mu_L-\epsilon_F|}{D}+\frac{\Gamma_R}{\Gamma}\ln\frac{|\Omega+\mu_R-\epsilon_F|}{D}\right] \tag{A7}$$

$$\Pi''_R(\Omega) = \frac{-N\Gamma\tilde{\Gamma}}{\pi\epsilon_F}\left[\frac{\Gamma_L}{\Gamma}\left(\frac{1}{1-\frac{\Omega+\mu_L}{\epsilon_F}} - \frac{1}{1-\frac{\mu_L}{\epsilon_F}}\right) + \frac{\Gamma_R}{\Gamma}\left(\frac{1}{1-\frac{\Omega+\mu_R}{\epsilon_F}} - \frac{1}{1-\frac{\mu_R}{\epsilon_F}}\right)\right]. \tag{A8}$$

Similarly, to $\mathcal{O}(1/N)$, Π^K is given by

$$\Pi^K(\Omega) = \frac{-2iN\Gamma\tilde{\Gamma}}{\pi\epsilon_F}\left[\frac{\Gamma_L^2}{\Gamma^2}\mathrm{sgn}(\Omega)\left(\frac{1}{1-\frac{\Omega+\mu_L}{\epsilon_F}} - \frac{1}{1-\frac{\mu_L}{\epsilon_F}}\right) + \frac{\Gamma_R^2}{\Gamma^2}\mathrm{sgn}(\Omega)\left(\frac{1}{1-\frac{\Omega+\mu_R}{\epsilon_F}} - \frac{1}{1-\frac{\mu_R}{\epsilon_F}}\right) + \frac{\Gamma_L\Gamma_R}{\Gamma^2}\mathrm{sgn}(\Omega+\mu_R-\mu_L)\left(\frac{1}{1-\frac{\Omega+\mu_R}{\epsilon_F}} - \frac{1}{1-\frac{\mu_L}{\epsilon_F}}\right) + \frac{\Gamma_L\Gamma_R}{\Gamma^2}\mathrm{sgn}(\Omega+\mu_L-\mu_R)\left(\frac{1}{1-\frac{\Omega+\mu_L}{\epsilon_F}} - \frac{1}{1-\frac{\mu_R}{\epsilon_F}}\right)\right]. \tag{A9}$$

The bosonic propagators may be evaluated from the Dyson equation

$$D^{-1}_{b,b^*} = D_0^{-1} - \Pi - \delta\Pi^{(1)}. \tag{A10}$$

To $\mathcal{O}(1/N)$, $\delta\Pi^{(1)}$ does not contribute. So we have

$$D^{R/A,-1}_{b,b^*} = D_0^{-1} - \Pi^{R/A} \tag{A11}$$

$$D^K_{b,b^*} = D^R_{b,b^*}\Pi^K D^A_{b,b^*}. \tag{A12}$$

Evaluating the above, we get

$$\mathrm{Re}[D^R_{b,b^*}(\Omega)] = \frac{1}{\Omega - \epsilon_F + E_0 - \frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\frac{|\Omega+\mu_L-\epsilon_F|}{D} + \frac{\Gamma_R}{\Gamma}\ln\frac{|\Omega+\mu_R-\epsilon_F|}{D}\right]}. \tag{A13}$$

Using the saddle-point equation Eq. (35), the above becomes

$$\mathrm{Re}[D^R_{b,b^*}(\Omega)] = \frac{1}{\Omega - \frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\left|\frac{\Omega+\mu_L-\epsilon_F}{\mu_L-\epsilon_F}\right| + \frac{\Gamma_R}{\Gamma}\ln\left|\frac{\Omega+\mu_R-\epsilon_F}{\mu_R-\epsilon_F}\right|\right]}. \tag{A14}$$

Similarly, the imaginary part to $O(1/N)$ (where $D^R = Re[D^R] + i\,\mathrm{Im}[D^R]$) is

$$\mathrm{Im}[D^R_{b,b^*}(\Omega)] = \frac{\frac{-N\Gamma\tilde{\Gamma}}{\pi\epsilon_F}\left[\frac{\Gamma_L}{\Gamma}\left(\frac{1}{1-\frac{\Omega+\mu_L}{\epsilon_F}} - \frac{1}{1-\frac{\mu_L}{\epsilon_F}}\right) + \frac{\Gamma_R}{\Gamma}\left(\frac{1}{1-\frac{\Omega+\mu_R}{\epsilon_F}} - \frac{1}{1-\frac{\mu_R}{\epsilon_F}}\right)\right]}{\left(\Omega - \frac{N\Gamma}{\pi}\left[\frac{\Gamma_L}{\Gamma}\ln\left|\frac{\Omega+\mu_L-\epsilon_F}{\mu_L-\epsilon_F}\right| + \frac{\Gamma_R}{\Gamma}\ln\left|\frac{\Omega+\mu_R-\epsilon_F}{\mu_R-\epsilon_F}\right|\right]\right)^2}. \tag{A15}$$

In the evaluation of the spectral density, we also need the anomalous boson propagators $D^{R,A,K}_{b,b}, D^{R,A,K}_{b^*,b^*}$. To $\mathcal{O}(\frac{1}{N})$, we find

$$D^R_{b,b}(\Omega) = D^R_{b^*,b^*}(\Omega) = 2D^R_{b,b^*}(\Omega)\,\delta\Pi^{R(2)}(\Omega)D^R_{b,b^*}(-\Omega), \tag{A16}$$

$$D^A_{b,b}(\Omega) = D^A_{b^*,b^*}(\Omega) = 2D^A_{b,b^*}(\Omega)\,\delta\Pi^{A(2)}(\Omega)D^A_{b,b^*}(-\Omega), \tag{A17}$$

$$D^K_{b,b}(\Omega) = D^K_{b^*,b^*}(\Omega) = 2D^R_{b,b^*}(\Omega)\,\delta\Pi^{K(2)}(\Omega)D^A_{b,b^*}(-\Omega) \tag{A18}$$

with $\delta\Pi^{R,A,K(2)}$ defined in Eqs. (67)–(69).

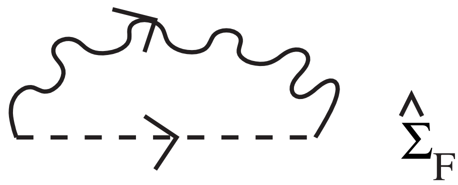

FIG. 6. Fermionic self-energy.

APPENDIX B: EVALUATION OF FERMIONIC SELF-ENERGY

These are defined as

$$\Sigma_F^{R,A}(1,2) = \frac{i}{2}[D_{b,b^*}^{R,A}(1,2)\Sigma_c^K(1,2) + D_{b,b^*}^K(1,2)\Sigma_c^{R,A}(1,2)], \tag{B1}$$

$$\Sigma_F^K(1,2) = \frac{i}{2}[D_{b,b^*}^K(1,2)\Sigma_c^K(1,2) + D_{b,b^*}^R(1,2)\Sigma_c^R(1,2) + D_{b,b^*}^A(1,2)\Sigma_c^A(1,2)] \tag{B2}$$

and are represented by the diagram in Fig. 6.

APPENDIX C: EXPANSION COEFFICIENTS IN THE EXPRESSION FOR THE SUSCEPTIBILITY AND THE CONDUCTANCE

The expansion coefficients in Eq. (141) are given by

$$C_{S0} = m(-L_2^{\text{eq}} - L_1^{\text{eq}} + 2L_3^{\text{eq}} - mJ_0^{\text{eq}}), \tag{C1}$$

$$C_{S1} = -6mL_3^{\text{eq}} + 6mL_4^{\text{eq}} - m^2\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^2}\frac{x^2(x+6)}{(1+x)^4} + m^3\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^3}\frac{2x^3}{(1+x)^4}, \tag{C2}$$

$$C_{S2} = mL_2^{\text{eq}} - 3mL_4^{\text{eq}} - 7mL_3^{\text{eq}} - 3mL_1^{\text{eq}} + 12mL_5^{\text{eq}}, \tag{C3}$$

$$-m^2\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^2}\frac{x^2(x^2+11x+20)}{2(1+x)^5} + m^3\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^3}\frac{x^3(x+2)}{(1+x)^5}, \tag{C4}$$

$$C_{S3} = 10mL_3^{\text{eq}} + 3mL_1^{\text{eq}} - 12mL_4^{\text{eq}} - mL_2^{\text{eq}} + m^2\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^2}\frac{x^2(3x-5)}{(1+x)^5} + m^3\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^3}\frac{x^3(x+9)}{(1+x)^5}, \tag{C5}$$

$$-m^4\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^4}\frac{3x^4}{(1+x)^5}. \tag{C6}$$

Whereas the coefficient C_{G1} appearing in the expression for the conductance in Eq. (166) is

$$C_{G1} = -2mL_2^{\text{eq}} + 8mL_3^{\text{eq}} - 6mL_4^{\text{eq}} - m^2\int_0^{D/T_A}\frac{dx}{[x+m\ln(1+x)]^2}\frac{2x^3}{(1+x)^4}. \tag{C7}$$

APPENDIX D: COMPUTATION OF THE SPECTRAL DENSITY

In this section we give explicit expressions for each of the five diagrams that contribute to the spectral density,

$$\text{Im}[G_{f_m,b}^R] = T_a + T_b + T_c + T_d + T_e \tag{D1}$$

where T_i is the ith diagram in Fig. 5. We find

$$T_a + T_b + T_c = -i\frac{\Gamma}{T_A^2\left(1-\frac{\Omega}{T_A}\right)^2}\frac{1}{(1+m_V)^2}\left[1 + \frac{2}{N}\left(\frac{-m^2}{1+m_V}L_2 + \frac{m^3}{1+m_V}k + \frac{m^3}{(1+m_V)^2}L_1\sum_i\frac{\Gamma_i/\Gamma}{(1-\mu_i/T_A)^2} - \frac{m^2}{1+m_V}\frac{L_1}{1-\Omega/T_A}\right)\right], \tag{D2}$$

$$+\frac{i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{-(\mu_\alpha-\mu_\beta)/T_A}^{D/T_A}dx\left[\frac{1}{\left(1+x-\frac{\Omega}{T_A}\right)^2} - \frac{1}{\left(1-\frac{\Omega}{T_A}\right)^2}\right]\frac{\left(\frac{1}{1+x-\frac{\mu_\beta}{T_A}} - \frac{1}{1-\frac{\mu_\alpha}{T_A}}\right)}{\left[x + m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]^2}, \tag{D3}$$

$$
-\frac{i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\left\{\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{(\Omega-\mu_\alpha)/T_A}^{D/T_A}dx\frac{1}{\left(1+x-\frac{\Omega}{T_A}\right)^2}\frac{\left(\frac{1}{1+x-\frac{\mu_\beta}{T_A}}-\frac{1}{1-\frac{\mu_\beta}{T_A}}\right)}{\left[x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]^2}\right\}, \tag{D4}
$$

$$
-\frac{i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)^2}\left\{\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{-(\Omega-\mu_\alpha)/T_A}^{D/T_A}dx\frac{\left(\frac{1}{1+x-\frac{\mu_\beta}{T_A}}-\frac{1}{1-\frac{\mu_\beta}{T_A}}\right)}{\left[x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]^2}\right\}, \tag{D5}
$$

$$
-\frac{2i\pi}{N^2T_A}\frac{m^2}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)^3}\sum_\alpha\frac{\Gamma_\alpha}{\Gamma}\int_{-(\Omega-\mu_\alpha)/T_A}^{D/T_A}dx\frac{1}{x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)}. \tag{D6}
$$

Note that the expressions (D4)–(D6) are logarithmically divergent. These divergences will be canceled by terms in diagrams (d) and (e), as we show below. In particular

$$
T_d=\frac{2i\pi}{N^2T_A}\frac{m^2}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)}\sum_\alpha\frac{\Gamma_\alpha}{\Gamma}\int_{(\Omega-\mu_\alpha)/T_A}^{D/T_A}dx\frac{1}{\left(1+x-\frac{\Omega}{T_A}\right)^2}\frac{1}{x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)}, \tag{D7}
$$

$$
+\frac{2i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)}\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{-(\mu_\beta-\mu_\alpha)/T_A}^{D/T_A}dx\left(\frac{1}{1+x-\frac{\Omega}{T_A}}\right)\frac{\left(\frac{1}{1+x-\frac{\mu_\alpha}{T_A}}-\frac{1}{1-\frac{\mu_\beta}{T_A}}\right)}{\left[x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]^2}. \tag{D8}
$$

Note that divergence in Eq. (D7) cancels the divergence in Eq. (D6). Moreover, we find

$$
\begin{aligned}
T_e=&\frac{-2i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)}\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{-(\mu_\alpha-\mu_\beta)/T_A}^{D/T_A}dx\left[\frac{1}{x\left(1-x-\frac{\Omega}{T_A}\right)}\right]\\
&\times\left\{\frac{\ln\left(1+\frac{x}{1-\frac{\mu_\beta}{T_A}}\right)+\ln\left(1-\frac{x}{1-\frac{\mu_\alpha}{T_A}}\right)}{\left[x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]\left[x-m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1-\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]}\right\},
\end{aligned} \tag{D9}
$$

$$+\frac{2i\pi}{N^2T_A}\frac{m^3}{(1+m_V)^2}\frac{1}{\left(1-\frac{\Omega}{T_A}\right)}\sum_{\alpha\beta}\frac{\Gamma_\alpha\Gamma_\beta}{\Gamma^2}\int_{-(\Omega-\mu_\alpha)/T_A}^{D/T_A}dx\left[\frac{1}{x\left(1-x-\frac{\Omega}{T_A}\right)}\right]$$
$$\times\left\{\frac{\ln\left(1+\frac{x}{1-\frac{\mu_\beta}{T_A}}\right)+\ln\left(1-\frac{x}{1-\frac{\mu_\beta}{T_A}}\right)}{\left[x+m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1+\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]\left[x-m\sum_i\frac{\Gamma_i}{\Gamma}\ln\left(1-\frac{x}{1-\frac{\mu_i}{T_A}}\right)\right]}\right\}. \tag{D10}$$

Note that Eq. (D10) cancels the divergence in Eqs. (D4) and (D5), while the divergence in Eq. (D8) is canceled by that in Eq. (D9).

APPENDIX E: FAILURE OF EXTRAPOLATION TO $N=2$ FOR SYSTEMS IN EQUILIBRIUM

Many N-fold degenerate magnetic impurity models, besides being amenable to $1/N$ perturbative approaches, are also exactly solvable by Bethe-Ansatz. However, a comparison between $1/N$ results and exact solutions are not straightforward as the two approaches use different cutoff schemes (for discussion on this see Ref. 17). Therefore the quantities that may be easily compared are universal quantities that are independent of the cutoff and T_K. One such quantity is the Wilson ratio $R=\frac{\pi^2k_B^2}{J(J+1)g^2\mu_B^2}\frac{\chi_S}{\gamma}$, where χ_S is the impurity susceptibility and $\gamma=-\frac{\partial^2F}{\partial T^2}$ is the specific-heat coefficient. The exact solution of the Coqblin-Schrieffer Hamiltonian gives[28]

$$R=\frac{N}{N-1}. \tag{E1}$$

Thus, for $N=2$ and $R=2$. On the other hand a perturbative $1/N$ expansion gives[29] $R=1+\frac{1}{N}$. Clearly, setting $N=2$ in this expression gives the rather incorrect result of $R=1.5$, showing that a naive extrapolation of the results of large N to the case of $N=2$ does not work. Another example is the value of the zero-bias conductance through a N-fold degenerate level in a quantum dot. The exact answer is

$$G=N\frac{e^2}{h}\frac{4\Gamma_L\Gamma_R}{(\Gamma_L+\Gamma_R)^2}\sin^2\frac{\pi n_F}{N}, \tag{E2}$$

n_F being the average charge on the level which approaches the value $n_F=1$ in the Kondo limit. Therefore, when $\Gamma_L=\Gamma_R$ and $N=2$, the conductance in the Kondo limit reaches the maximum possible value of $2e^2/h$. For $N\gg1$, a $1/N$ expansion gives $G=\frac{e^2}{h}\frac{\pi^2}{N}$, where again a naive substitution of $N=2$ gives the incorrect result of $G=\frac{e^2}{h}\frac{\pi^2}{2}$.

The extrapolation besides giving incorrect numerical values often does not capture the qualitative behavior of the temperature dependence of various observables. As an example let us consider the conductivity for the infinite-U Anderson model for a bulk system (rather than a quantum dot). If $\tau^{-1}(\omega,T)=i\,\mathrm{Im}[G_{f,b}]$ is the scattering rate due to the impurity, then the conductivity in a bulk geometry is $\sigma^{\mathrm{bulk}}\sim\int d\omega(-\frac{\partial f}{\partial\omega})\tau(\omega,T)$, whereas the conductance in a quantum-dot geometry (as has been considered in this paper) is $G\sim\int d\omega(-\frac{\partial f}{\partial\omega})\tau^{-1}(\omega,T)$. The exact answer for a $N=2$ bulk system is

$$\frac{\sigma^{\mathrm{bulk}}}{\sigma^{\mathrm{bulk}}(T=0)}=1+c_T\left(\frac{T}{\tilde{T}_K}\right)^2 \tag{E3}$$

where c_T is a positive coefficient. On the other hand a $1/N$ result for the temperature-dependent conductivity for the infinite-U Anderson model is[22]

$$\frac{\sigma^{\mathrm{bulk}}}{\sigma^{\mathrm{bulk}}(T=0)}=1+\pi^2\left(\frac{T}{T_K}\right)^2\left[1-\frac{8}{3N}\right]. \tag{E4}$$

Setting $N=2$ in the above equation gives a qualitatively different result from Eq. (E3) as it predicts that the conductivity will decrease with temperature (rather than increase).

The above discussion shows that large-N results cannot be used to extrapolate to $N=2$. However, comparison with exact Bethe-Ansatz solutions[17] shows that large-N works well for $N\geq4$.

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